A Queue-Based Model for Binary Communication Channels*

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Abstract

A model for a binary additive noise communication channel with memory is introduced. The channel noise process, which is generated according to a ball sampling mechanism involving a queue of finite length M, is a stationary ergodic M'th order Markov source. The channel properties are analyzed and several of its statistical and information theoretical quantities (e.g., block transition distribution, autocorrelation function, capacity) are derived in close form. The capacity of the queue-based channel is also analytically and numerically compared for a variety of channel conditions with the capacity of other binary models, such as the finite-memory contagion channel, the Gilbert-Elliott channel and the Fritchman channel.

Keywords: Binary channels with memory, finite-state channels, additive bursty noise, Markov sources, capacity, autocorrelation function, error statistics.

1 Introduction

Most real world communication channels are known to experience fading and noise distortions in a bursty fashion. In order to design effective communication systems for such channels, it is critical to fully understand their behavior. This is achieved via channel modeling, where the primary objective is to provide a model whose properties are both complex enough to closely capture the real channel statistical characteristics, and simple enough to allow mathematically tractable system analysis.

The most commonly used models to represent the discretized version (under hard-decision demodulation) of binary-input fading channels with memory are the Gilbert-Elliott channel (GEC) [4, 2] and the Fritchman channel (FC) [3]. These models, which have been partly adopted for historical reasons (as they were introduced in the 1960s), are described by binary additive error sources generated via finite-state hidden Markov models (HMMs)[1]. Due to their HMM structure, such channels are often difficult to mathematically analyze (particularly when incorporated within an overall source and/or channel coded system) since they do not admit exact closed-form expressions for their

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[1] A description of other lesser known, but related, finite or infinite state HMM based channel models is provided in [10].

block transition distribution and capacity. In [1, Section VI], Alajaji and Fuja proposed a simple binary additive channel with memory, referred to as the finite memory contagion channel (FMCC), where the noise process is generated via a finite-memory version of Polya's contagion urn scheme [9]. The resulting channel has a stationary ergodic M'th order Markov noise source and is fully described by only three parameters. Furthermore, unlike the GEC and FC models, it admits single-letter analytical expressions for its block transition distribution and capacity, which is an attractive feature for mathematical analysis. This model has recently been adopted in several joint source-channel coding studies (e.g., [5, 11]) where the channel statistics are incorporated into the system design in order to fully exploit the channel capacity (which is higher than the capacity of the traditionally used equivalent memoryless channel achieved via ideal interleaving).

In this work, we introduce a new binary additive noise channel with memory based on a finite queue of length M. The channel model, of which a simplified version was studied in [12], has also an M'th order Markov noise source that is fully characterized by four parameters, making it more sophisticated than the FMMC for channel modeling (as it has an additional degree of freedom) while remaining mathematically tractable. Indeed, it can be shown that the FMMC is a *special case* of our proposed queue-based channel (QBC) under identical channel conditions (see Section 3). It is also important to point out that in a recent work [7, 8], Pimentel *et. al.* showed (numerically) that the class of binary channel models with additive M'th order Markov noise (to which both the QBC and FMMC models belong) is a good approximation[2], in terms of autocorrelation function and variational distance, to the family of hard-decision frequency-shift keying demodulated correlated Rayleigh and Rician fading channels for a broad range of fading environments.

The rest of this paper is organized as follows. In Section 2, we investigate the statistical properties of the QBC model and derive its stationary distribution, block transition probability, capacity and autocorrelation function. In Section 3, the QBC is compared analytically, in terms of capacity, with the FMCC and a particular class of the Fritchman channel for the same bit error rate (BER), correlation coefficient and memory order M. Finally, numerical results and discussions are presented in Section 4.

2 Queue-Based Channel with Memory

We first present the QBC model described by: $Y_n = X_n \oplus E_n$, for $n = 1, 2, 3, \cdots$, where the random variables X_n, E_n, and Y_n are, respectively, the nth input, noise, and output of the channel, and where $\oplus$ denotes addition modulo 2. It is assumed that the input and error sequences are independent from each other. The noise process is generated according to the following mechanism. Consider the following two parcels.

- **Parcel 1** is a queue of length M, that contains *initially* M balls, as shown in Fig. 1.

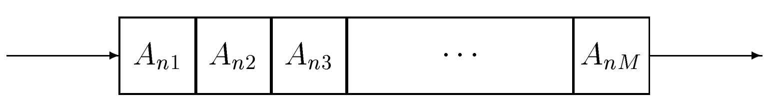

Figure 1: A queue of length M.

[2] This class of channels is also shown to be a better approximation than the GEC model, particularly under Rician fading.

The random variables A_{nk} (n is a time index referring to the nth experiment), $k = 1, 2, \cdots, M$, represent the color of the ball in the corresponding cell of the queue at time n:

$$A_{nk} = \begin{cases} 1, & \text{if the } k\text{th cell contains a red ball,} \\ 0, & \text{if the } k\text{th cell contains a black ball.} \end{cases}$$

- **Parcel 2** is an urn that contains a very large number of balls where the proportion of black balls is $1 - p$ and the proportion of red balls is p, where $p \in (0, 1)$; usually $p \ll 1/2$.

We assume that the probability of selecting parcel 1 (the queue) is ε, while the probability of selecting parcel 2 (the urn) is $1 - \varepsilon$ and $\varepsilon \in (0, 1)$. The error process $\{E_n\}_{n=1}^{\infty}$ is generated according to the following procedure. By flipping a biased coin (with Pr(Head)=ε), we select one of the two parcels (select the queue if Heads and the urn if Tails). If parcel 2 (the urn) is selected, a pointer randomly points at a ball, and identifies its color. If parcel 1 (the queue) is selected, the procedure is determined by the length of the queue. If $M \geq 2$, a pointer points at the ball in cell k with probability $1/(M - 1 + \alpha)$, for $k = 1, 2, \cdots, M - 1$ and $\alpha \geq 0$, and points at the ball in cell M with probability $\alpha/(M - 1 + \alpha)$, and identifies its color. If $M = 1$, a pointer points at the ball in the only cell of the queue with probability 1; i.e., $\alpha = 1$.

- If the selected ball is red, we introduce a red ball in cell 1 of the queue, pushing the last ball in cell M out.
- If the selected ball is black, we then introduce a black ball in cell 1 of the queue, pushing the last ball in cell M out.

The error process $\{E_n\}_{n=1}^{\infty}$ is then modeled as follows:

$$E_n = \begin{cases} 1, & \text{if the } n\text{th experiment points at a red ball,} \\ 0, & \text{if the } n\text{th experiment points at a black ball.} \end{cases}$$

We define the state of the channel to be $\underline{S}_n \stackrel{\triangle}{=} (A_{n1}, A_{n2}, \cdots, A_{nM})$, the binary $M-$tuple in the queue after the nth experiment is completed. Note that, in terms of the error process, the channel state at time n can be written as $\underline{S}_n = (E_n, E_{n-1}, \cdots, E_{n-M+1})$, for $n \geq M$.

2.1 Properties of the Noise Process

We now investigate the properties of the binary error process $\{E_n\}_{n=1}^{\infty}$. We first observe that, for $n \geq M + 1$,

$$\begin{aligned} &\Pr{}^{(M)}(E_n = 1 \mid E_{n-1} = e_{n-1}, \cdots, E_1 = e_1) \\ &= \varepsilon \left(\frac{e_{n-1}}{M - 1 + \alpha} + \cdots + \frac{e_{n-M+1}}{M - 1 + \alpha} + \frac{e_{n-M} \cdot \alpha}{M - 1 + \alpha} \right) + (1 - \varepsilon)p \\ &= \Pr{}^{(M)}(E_n = 1 \mid E_{n-1} = e_{n-1}, \cdots, E_{n-M} = e_{n-M}), \qquad (1) \end{aligned}$$

where $e_l \in \{0, 1\}$, $l = 1, \cdots, n - 1$. Hence $\{E_n\}_{n=1}^{\infty}$ is a homogeneous (or time-invariant) Markov process of order M.

Throughout this work, we consider the case where the initial distribution of the Markov noise $\{E_n\}_{n=1}^{\infty}$ is drawn according to its stationary distribution; hence the error process $\{E_n\}_{n=1}^{\infty}$ is stationary. We also obtain that $\{\underline{S}_n\}_{n=1}^{\infty}$ is a homogeneous Markov process with stationary distribution $\boldsymbol{\pi}^{(M)} \triangleq (\pi_0^{(M)};\ \pi_1^{(M)};\ \cdots; \pi_i^{(M)};\ \cdots; \pi_{2^M-1}^{(M)})$.

If $p_{ij}^{(M)}$ denotes the transition probability that $\underline{S}_n$ goes from state i to state j, $i, j = 0, 1, \cdots, 2^M - 1$, the transition matrix of the process $\{\underline{S}_n\}_{n=1}^{\infty}$ can be written as

$$\boldsymbol{Q}_{\mathrm{QBC}}^{(M)} = \left[p_{ij}^{(M)}\right]$$

with

$$p_{ij}^{(M)} = \begin{cases} \frac{(M-\omega_i^{(M)}-1+\alpha)\varepsilon}{M-1+\alpha} + (1-\varepsilon)(1-p), & \text{if } j = \lfloor \frac{i}{2} \rfloor, \text{ and } i \text{ is even,} \\ \frac{(M-\omega_i^{(M)})\varepsilon}{M-1+\alpha} + (1-\varepsilon)(1-p), & \text{if } j = \lfloor \frac{i}{2} \rfloor, \text{ and } i \text{ is odd,} \\ \frac{\omega_i^{(M)}\varepsilon}{M-1+\alpha} + (1-\varepsilon)p, & \text{if } j = \lfloor \frac{i+2^M}{2} \rfloor, \text{ and } i \text{ is even,} \\ \frac{(\omega_i^{(M)}-1+\alpha)\varepsilon}{M-1+\alpha} + (1-\varepsilon)p, & \text{if } j = \lfloor \frac{i+2^M}{2} \rfloor, \text{ and } i \text{ is odd,} \\ 0, & \text{otherwise;} \end{cases} \tag{2}$$

where $\omega_i^{(M)}$ is the number of "ones" in the M-bit binary representation of the ith row in $\boldsymbol{Q}_{\mathrm{QBC}}^{(M)}$. We note that any state can reach any other state with positive probability in a finite number of steps; therefore the process $\underline{S}_n$ is irreducible (and hence ergodic [1]).

It can be shown by solving $\boldsymbol{\pi}^{(M)} = \boldsymbol{\pi}^{(M)}\boldsymbol{Q}_{\mathrm{QBC}}^{(M)}$ via induction, that the stationary distribution $\boldsymbol{\pi}^{(M)}$ of the process is

$$\pi_i^{(M)} = \frac{\prod_{j=0}^{M-\omega_i^{(M)}-1}\left[j\frac{Cor}{1-Cor} + (1-BER)\right]\prod_{j=0}^{\omega_i^{(M)}-1}\left(j\frac{Cor}{1-Cor} + BER\right)}{\prod_{j=0}^{M-1}\left(1 + j\frac{Cor}{1-Cor}\right)}, \tag{3}$$

for $i = 0, 1, 2, \cdots, 2^M - 1$, where $\omega_i^{(M)}$ is the number of "ones" in the binary representation of the decimal integer i when memory is M, $\prod_{j=0}^{a}(\cdot) \triangleq 1$ if $a < 0$. Furthermore, the channel BER and correlation coefficient (Cor) are respectively given by

$$BER = \Pr(E_i = 1) = \Pr(E_1 = 1) = p, \tag{4}$$

and

$$Cor = \frac{Cov(E_2, E_1)}{Var(E_1)} = \frac{\frac{\varepsilon}{M-1+\alpha}}{1 - \frac{(M-2+\alpha)\varepsilon}{M-1+\alpha}}. \tag{5}$$

Lemma 1 *The stationary distribution $\pi_i^{(M)}$ obey the following recursion:*

$$\pi_i^{(M)} = \pi_{2i}^{(M+1)} + \pi_{2i+1}^{(M+1)}, \quad \textit{for } i = 0, 1, 2, \cdots, 2^M - 1. \tag{6}$$

2.2 Block Transition Probability

For a given input block $\underline{X} = [X_1, \cdots, X_n]$ and a given output block $\underline{Y} = [Y_1, \cdots, Y_n]$, where n is the blocklength, it can be shown using the Markovian property of the noise and state sources that the block transition probability of the resulting binary channel is as follows.

- For blocklength $n \leq M$,

$$\Pr^{(M)}(\underline{E}=\underline{e}) = \frac{\prod_{j=0}^{n-d_1^n-1}[j\frac{Cor}{1-Cor}+(1-BER)]\prod_{j=0}^{d_1^n-1}[j\frac{Cor}{1-Cor}+BER]}{\prod_{j=0}^{n-1}[1+j\frac{Cor}{1-Cor}]}, \qquad (7)$$

where $d_1^n = e_n + \cdots + e_1$, and $\prod_{j=0}^{a}(\cdot) \stackrel{\triangle}{=} 1$ if $a<0$.

- For blocklength $n \geq M+1$,

$$\begin{aligned}\Pr^{(M)}(\underline{E}=\underline{e}) &= L^{(M)} \prod_{i=M+1}^{n} \left\{\frac{\frac{\left(d_{i-M+1}^{i-1}+\alpha e_{i-M}\right)Cor}{1-Cor}+BER}{1+(M-1+\alpha)\frac{Cor}{1-Cor}}\right\}^{e_i} \\ &\quad \left\{\frac{\frac{\left(M-1-d_{i-M+1}^{i-1}+\alpha(1-e_{i-M})\right)Cor}{1-Cor}+(1-BER)}{1+(M-1+\alpha)\frac{Cor}{1-Cor}}\right\}^{1-e_i}, \qquad (8)\end{aligned}$$

where

$$L^{(M)} = \frac{\prod_{j=0}^{M-1-d_1^M}[j\frac{Cor}{1-Cor}+(1-BER)]\prod_{j=0}^{d_1^M-1}(j\frac{Cor}{1-Cor}+BER)}{\prod_{j=0}^{M-1}(1+j\frac{Cor}{1-Cor})},$$

$\prod_{j=0}^{a}(\cdot) \stackrel{\triangle}{=} 1$ if $a<0$, $d_1^M = e_M + \cdots + e_1$, $d_{i-M+1}^{i-1} = e_{i-1} + \cdots + e_{i-M+1}$ ($d_a^b = 0$ if $a>b$), $e_{i-M} = x_{i-M} \oplus y_{i-M}$, and $e_i = x_i \oplus y_i$ for $i = M+1, \cdots, n$.

2.3 Channel Capacity

The QBC is a channel with stationary ergodic Markov additive noise of memory M. The capacity $C_{\text{QBC}}^{(M)}$ of the channel is positive since the noise entropy rate is bounded above by 1 for fixed M, ε, p and α. $C_{\text{QBC}}^{(M)}$ is given by,

$$\begin{aligned} C_{\text{QBC}}^{(M)} &= \lim_{n\to\infty}\sup_{\underline{X}} \frac{1}{n} I(\underline{X};\underline{Y}) \\ &= 1 - H^{(M)}(E_{M+1} \mid E_M, E_{M-1}, \cdots, E_1) \\ &= 1 - \sum_{\omega=0}^{M-1} \binom{M-1}{\omega} L_\omega^{(M)} h_b\Big[\frac{\omega\frac{Cor}{1-Cor}+BER}{1+(M-1+\alpha)\frac{Cor}{1-Cor}}\Big] \\ &\quad - \sum_{\omega=1}^{M} \binom{M-1}{\omega-1} L_\omega^{(M)} h_b\Big[\frac{(\omega-1+\alpha)\frac{Cor}{1-Cor}+BER}{1+(M-1+\alpha)\frac{Cor}{1-Cor}}\Big], \qquad (9)\end{aligned}$$

where $\prod_{j=0}^{a}(\cdot) \stackrel{\triangle}{=} 1$, if $a<0$, $h_b(\cdot)$ is the binary entropy function, and

$$L_\omega^{(M)} = \frac{\prod_{j=0}^{M-1-\omega}[j\frac{Cor}{1-Cor}+(1-BER)]\prod_{j=0}^{\omega-1}(j\frac{Cor}{1-Cor}+BER)}{\prod_{j=0}^{M-1}(1+j\frac{Cor}{1-Cor})}, \qquad (10)$$

which is not a function of α .

Theorem 1 *The capacity $C_{QBC}^{(M)}$ of the QBC increases as the parameter α increases for fixed M, BER and Cor, and converges to 1 as α approaches to infinity for all M, BER and $Cor \neq 0$.*

2.4 Autocorrelation Function

The autocorrelation function (ACF) of a binary stationary process $\{E_n\}_{n=1}^{\infty}$ is given by:

$$\begin{aligned} R[m] &= E\{E_i E_{i+m}\} = \Pr(E_i = 1, E_{i+m} = 1) \\ &= \sum_{e_{i+1}=0}^{1} \cdots \sum_{e_{i+m-1}=0}^{1} \Pr(E_i = 1, E_{i+1} = e_{i+1}, \cdots, E_{i+m-1} = e_{i+m-1}, E_{i+m} = 1), \end{aligned}$$

where $E\{X\}$ denotes the expected value of the random variable X. Using (7) and (8), the ACF of the QBC is expressed as follows.

- If $m \le M-1$,

$$R[m] = BER[Cor + BER(1 - Cor)]. \tag{11}$$

- If $m \ge M$, the ACF of the QBC can be obtained by the following recursion.

$$\begin{aligned} R[m] &= \frac{1 - Cor}{1 + (M-2+\alpha)Cor} BER^2 \\ &+ \frac{Cor}{1 + (M-2+\alpha)Cor} \left(\sum_{i=m-M+1}^{m-1} R[i] + \alpha R[m-M] \right). \end{aligned} \tag{12}$$

2.5 Uniform Queue-Based Channel with Memory

The uniform queue-based channel (UQBC) was investigated in [12]. Actually, it is a special case of the QBC by fixing $\alpha = 1$; i.e., the experiment operates on the cells of the queue with equal probability $1/M$. The block transition distribution and capacity of the UQBC can be obtained by setting $\alpha = 1$ in (7), (8), and (9) (see also [12]).

Lemma 2 *The UQBC with memory M and the QBC with memory $M+1$ and $\alpha = 0$ have identical block transition probability for fixed BER and Cor; therefore the two channels have identical capacity under the above conditions.*

Theorem 2 *The capacity $C_{QBC}^{(M)}$ of the QBC is non-decreasing in M for fixed BER, Cor and $0 \le \alpha \le 1$.*

Proof For fixed BER and Cor, the capacity of the QBC is a function of the memory order M and parameter α. Let $C_{\mathrm{QBC}}^{(M)}(\alpha)$ denote the capacity of the QBC. Thus, for $0 < \alpha < 1$, we have

$$\begin{aligned} C_{\mathrm{QBC}}^{(M)}(\alpha) &< C_{\mathrm{QBC}}^{(M)}(1) \quad \text{(by Theorem 1)} \\ &= C_{\mathrm{QBC}}^{(M+1)}(0) \quad \text{(by Lemma 2)} \\ &< C_{\mathrm{QBC}}^{(M+1)}(\alpha) \quad \text{(by Theorem 1).} \end{aligned}$$

$\square$

3 Comparisons with Other Channels with Memory

In this section, we compare in terms of capacity the QBC with the FMCC [1] and a particular symmetric class of the Fritchman channel [3] under identical channel parameters.

3.1 Comparison with the Finite-Memory Contagion Channel

By comparing the UQBC with the FMCC [1] in terms of block transition probability, the following theorem is obtained [12].

Theorem 3 *The UQBC and the FMCC are statistically identical; i.e., they have the same block transition probability for the same memory M, BER and Cor. Therefore the two channels have identical capacity under the above conditions.*

Using Theorem 3 and the results in [1], the following asymptotic expression for $C_{\text{UQBC}}^{(M)}$ can be established as M approaches infinity:

$$\lim_{M\to\infty} C_{\text{UQBC}}^{(M)} = 1 - \int_0^1 h_b(z) f_Z(z) dz, \tag{13}$$

where $h_b(\cdot)$ is the binary entropy function and $f_Z(z)$ is the beta probability density function with parameters $BER(1 - Cor)/Cor$ and $(1 - BER)(1 - Cor)/Cor$ (denoted by u and v respectively), i.e.,

$$f_Z(z) = \beta_{u,v}(z) = \frac{\Gamma(u+v)}{\Gamma(u)\,\Gamma(v)} (1-z)^{(u-1)} z^{(v-1)}, \qquad z \in (0,1),$$

where $\Gamma(\cdot)$ is the gamma function: $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ for $x > 0$. We also obtain by Theorem 2 that (13) is an upper bound to the capacity of the UQBC for a given M.

Corollary 1 *For the same M, BER and Cor,*

$$C_{QBC}^{(M)} < C_{FMCC}^{(M)} \ \ \textit{(when } 0 \le \alpha < 1\textit{)}, \tag{14}$$

and

$$C_{QBC}^{(M)} > C_{FMCC}^{(M)} \ \ \textit{(when } \alpha > 1\textit{)}, \tag{15}$$

3.2 Comparison with the Symmetric Fritchman Channel

We define the symmetric Fritchman channel with K good states and one bad state ((K, 1)SFC) by the following transition matrix on its states

$$\boldsymbol{P}_{(K,1)\text{SFC}} = \begin{bmatrix} p_{00} & (1-p_{00})/K & \cdots & (1-p_{00})/K \\ (1-p_{00})/K & p_{00} & \cdots & (1-p_{00})/K \\ & \vdots & & \\ (1-p_{00})/K & \cdots & p_{00} & (1-p_{00})/K \\ (1-p_{11})/K & \cdots & (1-p_{11})/K & p_{11} \end{bmatrix}, \tag{16}$$

where p_{ii} is the block transition probability staying in state i, $i = 0, 1$.

By comparing the UQBC with $M = 1$ with the (K, 1)SFC in terms of the probability of an arbitrary error sequence, we obtain the following theorem.

Theorem 4 *For the same BER and Cor, and for any $K = 1, 2, \cdots$, the (K, 1)SFC is statistically identical to the UQBC with memory $M = 1$. Hence $C_{(K,\ 1)SFC} = C_{UQBC}^{(M=1)} \le C_{UQBC}^{(M)} \le C_{QBC}^{(M)}$, $\forall\ M \ge 1$ and $\alpha \ge 1$.*

We can explain this result by observing that the good states have the same stationary distribution $\frac{(1-BER)}{K}$ and they have the same transition pattern. Hence the good states can be combined into one big good state with stationary distribution $(1 - BER)$; this makes (K, 1)SFC behave like the (1, 1)FC (or UQBC with memory 1).

4 Numerical Results and Discussion

We next numerically evaluate the capacity of the QBC, GEC and FC models in terms of BER and Cor. We calculate the capacities of the QBC for memory order $M = 2$ and $\alpha = 10$ using (9). We also compute the capacity of the UQBC for memory order $M = 1, 2, 5, 10$, and its asymptotic upper bound (as $M \to \infty$) (see (13)).

Since the GEC is described by four parameters, we fix $p_G = 0.00002$ and $p_B = 0.92$ (which make our target Cor range from 0.1 to 0.9 well defined) and calculate the capacity in terms of BER and Cor using the algorithm of [6].

In [3], an explicit expression for the capacity of the FC channel with a single-error state and K good states ($(K,\ 1)$FC) is provided (an explicit formula for FC channels with more than a single error state is not known in general). We employ this expression to compute the capacity of the (2, 1)FC with the transition probability matrix

$$\boldsymbol{P}_{(2,\ 1)\mathrm{FC}} = \begin{bmatrix} p_{00} & (1-p_{00})/2 & (1-p_{00})/2 \\ 0.1 & 0.5 & 0.4 \\ (1-p_{11})/2 & (1-p_{11})/2 & p_{11} \end{bmatrix}.$$

where p_{00} and p_{11} vary as BER and Cor vary.

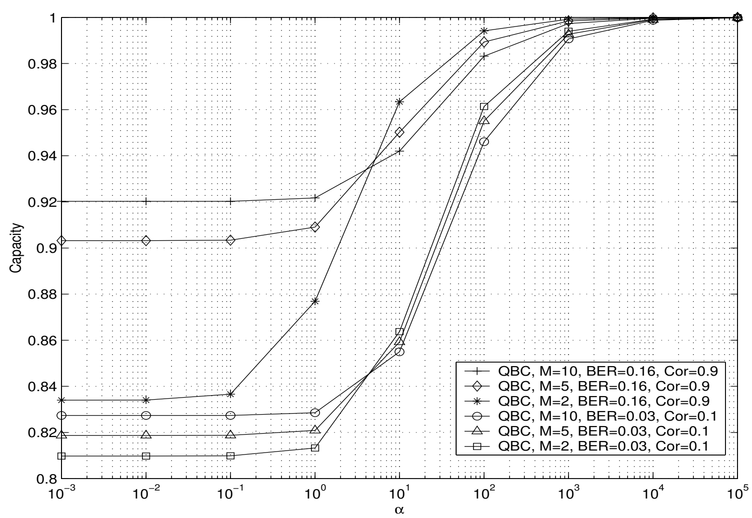

Figure 2: Capacity vs α for QBC.

Numerical capacity results for the above three channels are presented in Figs. 2 - 5. The effect of the cell parameter α on the capacity of the QBC is shown in Fig. 2; we note that for the same BER, Cor and memory order M, the capacity increases with α as predicted in Theorem 1. Furthermore, Fig. 2 illustrates Theorem 2 in the range $\alpha \leq 1$.

The results of Theorem 3, Corollary 1 and Theorem 4 are numerically illustrated in Figs. 3 - 5. We also note from the figures that the capacity of all channel models increases with decreasing BER and increasing Cor (as expected). We furthermore observe that, for the considered parameters, the QBC with $M = 2$ and $\alpha = 10$ has the biggest capacity, whereas the UQBC with $M = 1$ (or (1, 1)FC) provides the smallest capacity. When $Cor = 0.1$, the GEC and the UQBC with $M = 1$ have nearly equal capacities. For the same BER, the capacity of the QBC can be either smaller or bigger than that of the GEC and (2, 1)FC, depending on the values of Cor, M and α (see Fig. 5).

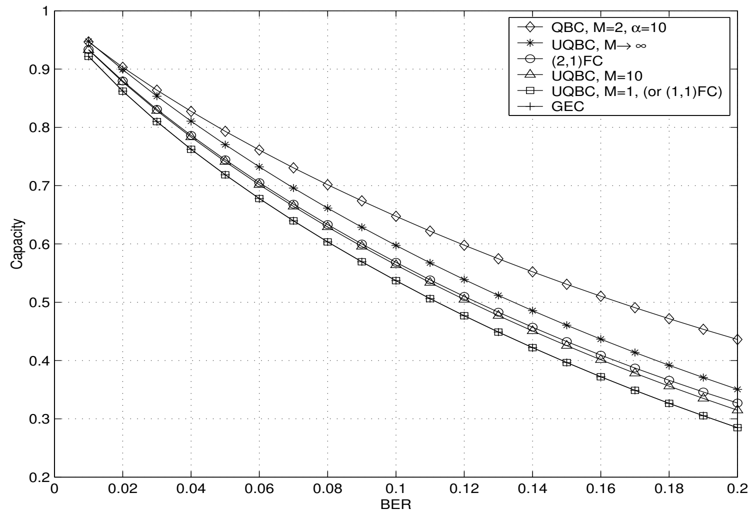

Figure 3: Capacity vs BER for Cor=0.1; $p_G = 0.00002$ and $p_B = 0.92$ (for GEC).

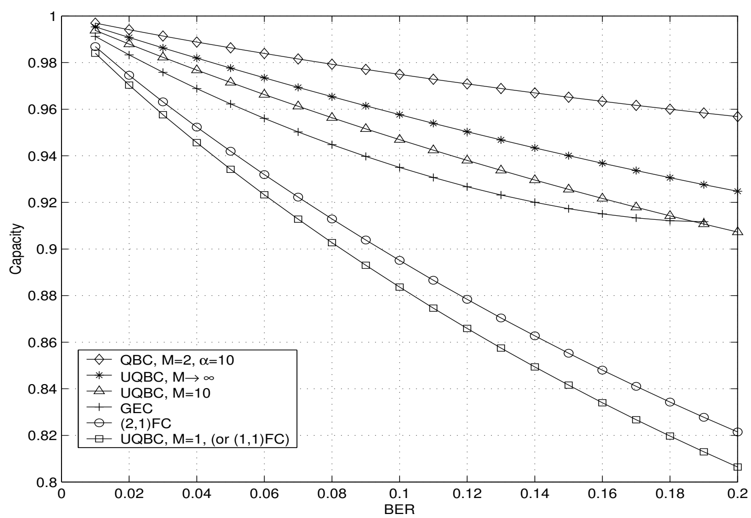

Figure 4: Capacity vs BER for Cor=0.9; $p_G = 0.00002$ and $p_B = 0.92$ (for GEC).

In conclusion, we point out that our QBC models enjoy the important feature of being able to characterize a wide class of binary communication channels with finite Markovian memory, while remaining mathematically simple and flexible (even for large values of the memory M). They hence provide an interesting and less complex alternative to the traditional GEC and Fritchman models.

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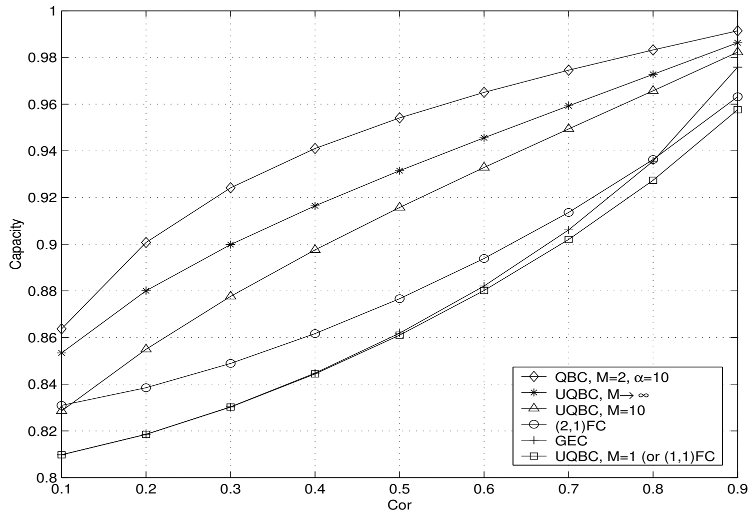

Figure 5: Capacity vs Cor for BER=0.03; $p_G = 0.00002$ and $p_B = 0.92$ (for GEC).

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