Soft-Decision COVQ Based on Turbo Codes

Guang-Chong Zhu and Fady Alajaji

Queen's University, Kingston, Ontario, Canada, K7L 3N6
Tel: (613) 533-2423, Fax: (613) 533-2964
E-mail: fady@shannon.mast.queensu.ca

Abstract: *A robust soft-decision channel optimized vector quantization (COVQ) scheme for Turbo-coded AWGN channels is proposed. The log-likelihood ratio (LLR) generated by the Turbo decoder is exploited in the COVQ design via the use of a q-bit scalar soft-decision demodulator. The concatenation of the Turbo encoder, modulator, AWGN channel, Turbo decoder, and q-bit soft-decision demodulator is modeled as a 2^{kr}-input, 2^{qkr}-output discrete memoryless channel (DMC), or a binary-input, 2^q-output DMC used kr times. A low-complexity COVQ scheme for this expanded discrete channel is then designed. Numerical results indicate substantial performance improvements over traditional tandem coding systems and COVQ schemes designed for hard-decision demodulated Turbo-coded channels ($q = 1$).*

Keywords: COVQ, Turbo codes, soft-decision demodulator, joint source-channel coding.

1. Introduction

Conventionally, source and channel coding have been designed separately (resulting in what we call a *tandem* coding system). As proved by Shannon, this separation of source and channel coding results in no loss of optimality provided unlimited coding delay and system complexity are allowed [17]. During the past few decades, significant improvements have been achieved in these two separate areas. One of the most noticeable techniques in fixed-rate source coding is source-optimized vector quantization (LBG-VQ) [13], while in channel coding, Turbo codes [5], [6] have been widely recognized as a major breakthrough due to their extraordinary performance. However, in practice, with constraints on delay and complexity, joint source-channel coding can significantly outperform traditional tandem coding systems (e.g., [1] – [4], [9], [11], [12], [14] – [16], [18], [19]).

In this work, we design and implement a *robust soft-decision* channel-optimized vector quantization (COVQ) scheme for Turbo-coded channels. More specifically, we employ the methods introduced in [1], [15] to design a COVQ system that improves the end-to-end performance by exploiting the log likelihood ratio (LLR) generated by the Turbo decoder. This is

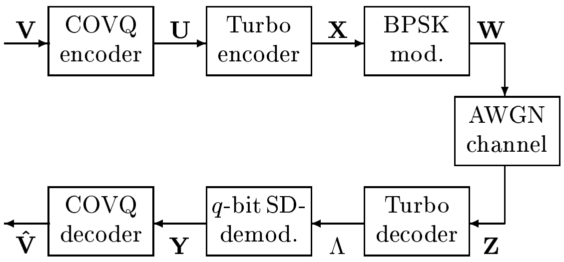

Figure 1: Block diagram of the system.

achieved via the use of a q-bit scalar soft-decision demodulator at the output of the Turbo decoder, and by designing a COVQ scheme for the resulting *expanded discrete channel* which consists of the concatenation of the Turbo encoded and decoded channel with the soft-decision demodulator. Alternative approaches for channel-optimized quantization using Turbo codes have been previously studied by Bakus and Khandani for scalar quantization [3], [4], and by Ho for vector quantization [11], where the *entire* (unquantized) soft-decision information provided by the LLR of the Turbo decoder is utilized. The performance of our scheme is comparable to Ho's, while the complexity is lower.

2. System design

The proposed system is as follows (see Figure 1). The COVQ encoder takes a k-dimensional real vector $\mathbf{V}$ as its input, operates at a rate of r bits per source sample, and generates kr bits as the output $\mathbf{U} \in \{0,1\}^{kr}$. This output is then fed into a Turbo code encoder, which is of rate R_c information bits/code bit. From the Turbo encoder, the output $\mathbf{X}$ is binary phase-shift keying (BPSK) modulated as $\mathbf{W} \in \{-1,+1\}^{kr/R_c}$ (assuming kr/R_c is an integer). The sequence $\{\mathbf{W}_i\}$ is then transmitted through an additive white Gaussian noise (AWGN) channel according to

$$Z_l = W_l + N_l, \qquad l = 1, 2, 3, \cdots,$$

where $\{N_l\}$ is an i.i.d. Gaussian noise source with zero mean and variance $N_0/2$.

At the receiver end, Turbo decoding is used to provide the LLR given by

$$\Lambda_l = \log \frac{Pr\{U_l = 1|\mathbf{Z}\}}{Pr\{U_l = 0|\mathbf{Z}\}}, \qquad l = 1, 2, 3, \cdots,$$

which is then demodulated via a q-bit uniform scalar quantizer $\alpha(\cdot)$ with quantization step Δ. The quantizer is described by

$$\alpha(\Lambda) = j \quad \text{if} \ \ \Lambda \in (T_{j-1}, T_j),$$

where $j = 0, 1, \cdots, 2^q - 1$.

The thresholds $\{T_j\}$ are uniformly spaced with quantization step Δ; they satisfy

$$T_j = \begin{cases} -\infty & \text{if} j = -1, \\ (j + 1 - 2^{q-1})\Delta, & \text{if} j = 0, 1, \cdots, 2^q - 2, \\ +\infty & \text{if} j = 2^q - 1. \end{cases}$$

Finally, these qkr bits are passed to the COVQ decoder, from which $\hat{\mathbf{V}}$, an estimation of $\mathbf{V}$, is produced.

3. Expanded DMC model

As observed by Berrou *et al.* [5], [6] and Colavolpe *et al.* [8], within a certain region of channel signal-to-noise ratio (CSNR), and for large information block length N, the LLR generated by the Turbo decoder is approximately Gaussian with mean $+M$ or $-M$, and variance σ_Λ^2. Therefore, the transition probability distribution for this equivalent channel can be approximated by

$$p(\Lambda_l | U_l = i) = \frac{1}{\sqrt{2\pi\sigma_\Lambda^2}} \mathrm{e}^{-\frac{(\Lambda_l - (2i-1)M)^2}{2\sigma_\Lambda^2}},$$

$$i = 0, 1, \qquad l = 1, 2, \cdots, kr.$$

The values of M and σ_Λ^2 depend on the structure of the Turbo encoder, the channel statistics, as well as the decoding algorithm used in the Turbo decoder. While analytical expressions for M and σ_Λ^2 are intractable, we obtain a reliable estimation of their values by data training. We then model the concatenation of the Turbo encoder, modulator, AWGN channel, Turbo decoder, and q-bit soft-decision demodulator as a 2^{kr}-input, 2^{qkr}-output discrete memoryless channel (DMC), or a binary-input, 2^q-output DMC used kr times.

For this channel model, if $\mathcal{U} = \{0, 1\}$ and $\mathcal{Y} = \{0, 1, 2, \cdots, 2^q - 1\}$, then the transition probability matrix Π is given by

$$\Pi = [\pi_{ij}], \qquad i \in \mathcal{U}, j \in \mathcal{Y}$$

where

Figure 2: Expanded DMC model of our system.

$$\begin{aligned} \pi_{ij} &= Pr\{Y = j | U = i\} \\ &= Pr\{\Lambda \in (T_{j-1}, T_j) | U = i\} \\ &= \frac{1}{\sqrt{2\pi\sigma_\Lambda^2}} \int_{T_{j-1}}^{T_j} \mathrm{e}^{-\frac{(\lambda - (2i-1)M)^2}{2\sigma_\Lambda^2}} d\lambda \\ &= \frac{1}{2} erfc\left(\frac{T_{j-1} - (2i-1)M}{\sqrt{2\sigma_\Lambda^2}}\right) \\ &\quad -\frac{1}{2} erfc\left(\frac{T_j - (2i-1)M}{\sqrt{2\sigma_\Lambda^2}}\right), \end{aligned}$$

where $erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \mathrm{e}^{-t^2}$ dt is the complimentary error function.

Hence, the channel transition probability matrix of our DMC model can be computed in terms of the quantization step Δ, the channel noise variance, and the complementary error function. It can be observed that this DMC is "weakly" symmetric in the sense that its transition probability matrix Π can be partitioned (along its columns) into symmetric arrays – where a symmetric array is defined as an array whose rows are permutations of each other, and whose columns are permutations of each other [10]; therefore, its capacity is achieved by a uniform input distribution. For each channel noise variance, the quantization step size Δ of the q-bit demodulator is chosen such that the capacity of this DMC is maximized.

Besides the DMC model above, we can also obtain the transition probability matrix Π via data training, where the expanded channel is regarded as a block-memoryless channel (with 2^{kr} inputs, and 2^{qkr} outputs); i.e., we ignore the memory from block-to-block and only consider the memory within a block (cf. [15]). This can be achieved by estimating the $2^{kr} \times 2^{qkr}$ transition probability matrix Π by using a long training sequence, and then using Blahut's algorithm [7] to calculate the channel capacity. The quantization step size Δ is chosen to maximize the channel capacity for each channel noise variance. This is a more accurate method since it captures the block memory in the expanded channel. Also, as kr increases, the model becomes more accurate.

4. COVQ design

We next design a COVQ for this 2^{kr}-input, 2^{qkr}-output DMC using the iterative algorithm described in [9]. Consider the COVQ system in Figure 2, which consists of an encoder mapping γ and a decoder mapping β. The encoder mapping is described by a partition $\mathcal{P} = \{S_{\mathbf{u}} \subset I\!R^k : \mathbf{u} \in \mathcal{U}^{kr}\}$ according to $\gamma(\mathbf{v}) = \mathbf{u}$ if $v \in S_{\mathbf{u}}, u \in \mathcal{U}^{kr}$, where $\mathbf{v} = (v_1, v_2, \cdots, v_k)$ is a block of k successive source samples. The DMC is described by its block channel transition matrix $P(\mathbf{y}|\mathbf{u})$, where $\mathbf{u} \in \mathcal{U}^{kr}$ and $\mathbf{y} \in \mathcal{Y}^{qkr}$. Finally, the decoder mapping β is given by a codebook $\mathcal{C} = \{\mathbf{c}_{\mathbf{y}} \in I\!R^k : \mathbf{y} \in \mathcal{Y}^{kr}\}$ according to $\beta(\mathbf{y}) = \mathbf{c}_{\mathbf{y}}, \mathbf{y} \in \mathcal{Y}^{kr}$.

The average squared-error distortion per sample is given by

$$D = \frac{1}{k} \sum_{\mathbf{u}} \int_{S_{\mathbf{u}}} f(\mathbf{v}) \left\{ \sum_{\mathbf{y}} P(\mathbf{y}|\mathbf{u}) \parallel \mathbf{v} - \mathbf{c}_{\mathbf{y}} \parallel^2 \right\} d\mathbf{v},$$

where $f(\mathbf{v})$ is the k-dimensional pdf of the source. For a fixed k and r, and a given source and channel, our goal is to minimize D by properly choosing $\mathcal{P}$ and $\mathcal{C}$.

For a given $\mathcal{C}$, the optimal partition $\mathcal{P}^* = \{S^*_{\mathbf{u}}\}$ is given by

$$\begin{aligned} S^*_{\mathbf{u}} &= \left\{ \mathbf{v} : \sum_{\mathbf{y}} P(\mathbf{y}|\mathbf{u}) \parallel \mathbf{v} - \mathbf{c}_{\mathbf{y}} \parallel^2 \right. \\ &\leq \left. \sum_{\mathbf{y}} P(\mathbf{y}|\tilde{\mathbf{u}}) \parallel \mathbf{v} - \mathbf{c}_{\mathbf{y}} \parallel^2 \quad \forall \tilde{\mathbf{u}} \in \mathcal{U}^{kr} \right\}, \end{aligned}$$

$u \in \mathcal{U}^{kr}$. On the other hand, for a given partition, the optimal codebook $\mathcal{C}^* = \{\mathbf{c}_{\mathbf{y}}{}^*\}$ is

$$\mathbf{c}_{\mathbf{y}}{}^* = \frac{\sum_u P(\mathbf{y}|\mathbf{u}) \int_{S_{\mathbf{u}}} \mathbf{v} f(\mathbf{v}) d\mathbf{v}}{\sum_u P(\mathbf{y}|\mathbf{u}) \int_{S_{\mathbf{u}}} f(\mathbf{v}) d\mathbf{v}}.$$

The codebook can be pre-computed off-line. Therefore, the COVQ decoding is implemented simply by a table-lookup with no extra computation. However, the memory for storing the codebook is high.

5. Numerical results and discussion

In Table 1, we present numerical results for the quantization of a Gauss-Markov source with correlation coefficient $\rho = 0.9$ over a BPSK-modulated AWGN channel used with Turbo codes. 80,000 training source vectors are used. The Turbo code is a rate-1/2, 16-state code with generator (37,21) and block length $N = 65536$ bits. A pseudo-random interleaver is used [6] and the number of decoding iterations is 10. The dimension of the COVQ source input is $k = 4$, the quantization rate is $r = 1$ bit/source symbol; therefore, with the rate $R_c = 1/2$ Turbo code, the overall rate is $r/R_c = 2$ channel symbols/source symbol. The channel signal-to-noise ratio (CSNR) is defined as

$$CSNR = \frac{E_s}{N_0/2} = \frac{1}{N_0/2},$$

where E_s is the symbol energy, and $E_s = R_c E_b$, where E_b is the bit energy. When $R_c = 1/2$,

$$CSNR = \frac{R_c E_b}{N_0/2} = \frac{E_b}{N_0}.$$

In comparison with a COVQ of rate $r = 2$ without using Turbo code, the performance of our scheme at $CSNR = 0.5$ dB is slightly worse than the COVQ scheme that does not employ Turbo codes (i.e., the scheme assigning all the available rate for source coding), since at this point the Turbo code bit error rate (BER) is high (above the 10^{-2} level). When CSNR is greater than 0.6 dB, our scheme with $q = 1$ (COVQ designed for the equivalent BSC model) offers superior performance over the COVQ scheme without Turbo codes; a slight increment of CSNR results in a drastic improvement of the performance. This is consistent with the fact that the BER performance curve of Turbo codes drops quickly around 0.7 dB. At 1.0 dB and above, the performance is very close to the theoretical limit.

Figure 3 shows the comparison of the performance generated by our scheme and other proposed schemes [11]. Our scheme offers comparable performance to Ho's system; however, the complexity of our scheme is lower. In comparison with the traditional tandem scheme (which consists of a noiseless LBG-VQ followed by a regular Turbo code), at $CSNR = 0$ dB, our scheme with $q = 4$ can achieve a gain of about 3 dB in signal-to-distortion ratio (SDR); at $CSNR = 0.6$ dB, the gain is about 3.5 dB. The performance is improved when q increases. For low CSNRs, the gain from $q = 1$ to $q = 4$ is as big as 1 dB. However, the most significant gain is achieved at $q = 2$. For high CSNRs, the gain due to increasing q is less obvious, since the performance is already very close to the theoretical limit.

6. Summary

In this work, we designed and implemented a COVQ scheme based on Turbo codes. The reliability information produced by the Turbo decoder was utilized via a q-bit scalar soft-decision demodulator. Within a certain region of the channel SNR, and with large block length N, the concatenation of the Turbo encoder, BPSK modulator, AWGN channel, Turbo decoder, and q-bit soft-decision demodulator was approximately modeled as a 2^{kr}-input, 2^{qkr}-output DMC. The COVQ scheme was designed for this expanded DMC. Significant improvements over the traditional tandem scheme was demonstrated. In comparison with the work by Ho [11], our scheme

Channel SNR	With TC	Without TC	
	$q=1$	$q=1$	$q=4$
0.5	3.10	3.97	5.16
0.6	4.95	4.03	5.25
0.7	7.57	4.09	5.33
0.8	7.79	4.16	5.41
0.9	7.83	4.22	5.49
1.0	7.85	4.28	5.57
1.1	7.86	4.34	5.66
1.2	7.87	4.41	5.74
∞	7.91	13.52	13.52

Table 1: SDR (in dB) performances of COVQ based on Turbo codes, (COVQ rate r=1 bit/sample, Turbo code rate R_c=1/2), compared with that of COVQ without Turbo codes, (COVQ rate r=2 bits/sample). Both use dimension k=2 Gauss-Markov Source (ρ=0.9), AWGN channel.

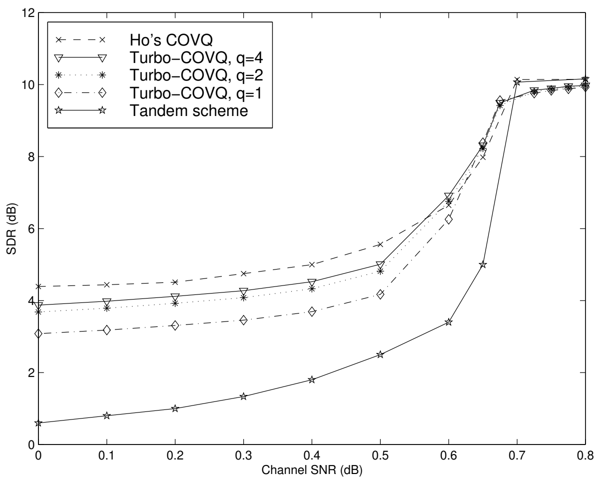

Figure 3: SDR as a function of channel SNR, k=4, r=1.

offered comparable performance, while the complexity was lower.

REFERENCES

[1] F. Alajaji and N. Phamdo, "Soft-decision COVQ for Rayleigh-fading channels," *IEEE Commun. Lett.*, Vol. 2, pp. 162–164, June 1998.

[2] E. Ayanoglu and R.M. Gray, "The design of joint source and channel trellis waveform coders," *IEEE Trans. Inform. Theory*, Vol. 33, pp. 855–865, Nov. 1987.

[3] J. Bakus and A.K. Khandani, "Combined source-channel coding using Turbo-codes," *Electron. Lett.*, Vol. 33, No. 19, pp. 1613–1614, Sept. 1997.

[4] J. Bakus and A.K. Khandani, "Quantizer design for channel codes with soft output decoding," submitted to *IEEE Trans. Inform. Theory.*

[5] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon limit error-correcting coding and decoding: Turbo-codes(1)," *Proc. 1993 IEEE Int. Conf. Commun.* Geneva, Switzerland, pp. 1064–1070, 1993.

[6] C. Berrou and A. Glavieux, "Near optimum error correcting coding and decoding: Turbo-codes," *IEEE Trans. Commun.*, Vol. 44, No. 10, pp. 1261–1271, Oct. 1996.

[7] R.E. Blahut, *Principles and Practice of Information Theory*, Addison Wesley, Massachusetts, 1988.

[8] G. Colavolpe, G. Ferrari, and R. Raheli, "Extrinsic information in Turbo decoding: a unified view," *Proc. of Globecom'99*, pp. 505–509, 1999.

[9] N. Farvardin and V. Vaishampayan, "On the performance and complexity of channel-optimized vector quantizers," *IEEE Trans. Inform. Theory*, pp. 155–160, Jan. 1991.

[10] R.G. Gallager, *Information Theory and Reliable Communication.* New York: Wiley, 1968.

[11] K.-P. Ho, "Soft-decoding vector quantizer using reliability information from Turbo-codes," *IEEE Commun. Lett.*, Vol. 3, pp. 208–210, July 1999.

[12] H. Kumazawa, M. Kasahara, and T. Namekawa, "A construction of vector quantizers for noisy channels," *Electronics and Engineering in Japan*, Vol. 67-B, pp. 39-47, Jan. 1984.

[13] Y. Linde, A. Buzo, and R.M. Gray, "An algorithm for vector quantizer design," *IEEE Trans. Commun.*, Vol. 28, pp. 84–95, 1980.

[14] F.-H. Liu, P. Ho and V. Cuperman, "Sequential reconstruction of vector quantized signals transmitted over Rayleigh fading channels," *Proc. of IEEE Int. Conf. Commun.*, pp. 23-27, New Orleans, 1994.

[15] N. Phamdo and F. Alajaji, "Soft-decision demodulation design for COVQ over white, colored and ISI Gaussian channels," *IEEE Trans. Commun.*, to appear.

[16] P.L. Secker and P.O. Ogunbona, "Methods of channel-optimized trellis source coding for the AWGN channel," *Proc. IEEE ISIT*, p. 236, Norway, June 1994.

[17] C.E. Shannon, "A mathematical theory of communications," *Bell Syst. Tech. J.*, Vol. 27, pp. 379–423, 1948.

[18] M. Skoglund and P. Hedelin, "Hadamard-based soft-decoding for vector quantization over noisy channels," *IEEE Trans. Inform. Theory*, Vol. 45, pp. 515–532, Mar. 1999.

[19] V. Vaishampayan and N. Farvardin, "Joint design of block source codes and modulation signal sets," *IEEE Trans. Inform. Theory*, Vol. 36, pp. 1230–1248, Jul. 1992.