Trellis-Coded Modulation and Transmit Diversity: Design Criteria and Performance Evaluation

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Abstract

This paper presents the selection criteria for trellis codes used with the simple transmit diversity scheme proposed in [1] ***and reviewed in this paper. The optimum metrics for soft decision Viterbi decoding are derived and the asymptotic coding gains are calculated. The design method achieves both coding gain and a diversity order of 2 using 2 transmit antennas and 1 receive antenna. It is also demonstrated that trellis codes optimized for AWGN are also optimal for use in conjunction with the transmit diversity scheme in Rayleigh fading.***

1. Introduction

The next generation wireless systems are supposed to have better quality and coverage, be more power and bandwidth efficient, and be deployed in diverse environments. Yet, the services have to remain affordable for widespread market acceptance. Inevitably, the next generation remote units must remain relatively simple. Fortunately, however, the economy of scale may allow more complex base stations. In fact, it appears that base station complexity may be the only plausible trade space for achieving the requirements of next generation wireless systems.

The fundamental phenomenon which makes reliable wireless transmission difficult is time-varying multipath fading [2]. It is this phenomenon which makes tetherless transmission a challenge when compared to fiber, coaxial cable, line-of-sight microwave or even satellite transmissions.

There are many techniques employed or proposed to combat or reduce the effect of multipath fading. However, most of these techniques are only effective in certain environments and deployment scenarios. Some of these techniques such as transmit power control are often not feasible because of cost or power constraints, and others such as spread spectrum communications are only effective when the channel exhibits certain characteristics. For instance, spread spectrum techniques reduce fading only when the channel has relatively large delay spread or small coherence bandwidth compared to the spreading bandwidth.

In most scattering environments, antenna diversity is a practical, effective and hence a widely applied technique for reducing the effect of multipath fading[2]. The classical approach is to use multiple antennas at the receiver and perform combining or selection and switching in order to improve the quality of the received signal. The major problem with using the receive diversity approach is the cost, size and power of the remote units. The use of multiple antennas and RF chains (or selection and switching circuits) makes the remote units larger and more expensive. As a result, diversity techniques have almost exclusively been applied to base stations to improve their reception quality. A base station often serves hundreds to thousands of remote units. It is therefore more economical to add equipment to base stations rather than the remote units. For this reason, transmit diversity schemes are very attractive. For instance, 1 antenna and 1 transmit chain may be added to a base station to improve the reception quality of all the remote units in that base stations's coverage area. The alternative is to add more antennas and receivers to all the remote units. The first solution is definitely more economical.

Recently, some interesting approaches for transmit diversity have been suggested. A delay diversity scheme was proposed by Wittneben [3][4] for base station simulcasting and later, independently, a similar scheme was suggested by Seshadri and Winters[5][6] for a single base station in which copies of the same symbol are transmitted through multiple antennas at different times hence creating an artificial multipath distortion. A maximum likelihood sequence estimator (MLSE) or a minimum mean squared error (MMSE) equalizer is then used to resolve multipath distortion and obtain diversity gain. Another interesting approach is space-time trellis coding introduced in [7], where symbols are encoded according to the antennas through which they are simultaneously transmitted, and are decoded using a maximum likelihood decoder. This scheme is very effective as it combines the benefits of FEC coding and diversity transmission to provide considerable performance gains. The cost for this scheme is additional processing which increases exponentially as a function of bandwidth efficiency (bits/sec/Hz) and the required diversity order. Therefore, for some applications it may not be practical or cost-effective.

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A simple and effective transmit diversity scheme was proposed in [1]. This scheme has a very simple implementation and provides the same diversity order as maximal-ratio receiver combining (MRRC). Using 2 transmit antennas and M receive antenna, the scheme provides a diversity order of $2M$ at the receiver. The scheme does not require any feedback from the receiver to the transmitter and does not require channel reciprocity; i.e., works in both TDD and FDD systems.

In this paper, we first review the simple transmit diversity scheme proposed in [1][1] . We then establish the design criteria for trellis codes used in conjunction with the transmit diversity scheme. Moreover, we provide the optimal soft-decision decoding metric for the trellis codes. We prove that the scheme provides a diversity order of 2 and further coding gain comparable to that of the trellis codes in the AWGN channel. It is shown that optimal trellis codes for AWGN channels are also optimal for the transmit diversity scheme in Rayleigh fading. By optimal we refer to the error event probability and not bit-error-rate (BER). Since Ungerboeck codes [10] are known to be optimal trellis codes in AWGN, we conclude that they are also optimal in Rayleigh fading with the transmit diversity scheme and may hence be used together with the transmit diversity scheme to provide both diversity and significant coding gain.

The outline of the paper is as follows: the transmit diversity scheme is reviewed in Section 2. In Section 3, the system model for the transmit diversity scheme with trellis coding is presented. In Section 4, the error performance of the scheme in analyzed, and the conclusions are made in Section 5.

2. A Review of the Transmit Diversity Scheme

In this section, we present a review of the transmit diversity scheme first published in [1]. For illustration, we describe the scheme with 2 transmit antennas and 1 receive antenna and show that it provides a diversity order of 2. The scheme may easily be generalized to 2 transmit antennas and M receive antennas to provide a diversity order of $2M$.

Figure 1 shows the baseband representation of the transmit diversity scheme with 2 transmit antennas and 1 receive antenna.

The new scheme is defined by three functions:

- the encoding and transmission sequence of information symbols at the transmitter
- the combining scheme at the receiver
- the decision rule for maximum likelihood detection.

1. The reference [1] may not be available at the time this paper is published. Therefore, we have reviewed the pertinent sections of [1] in this paper.

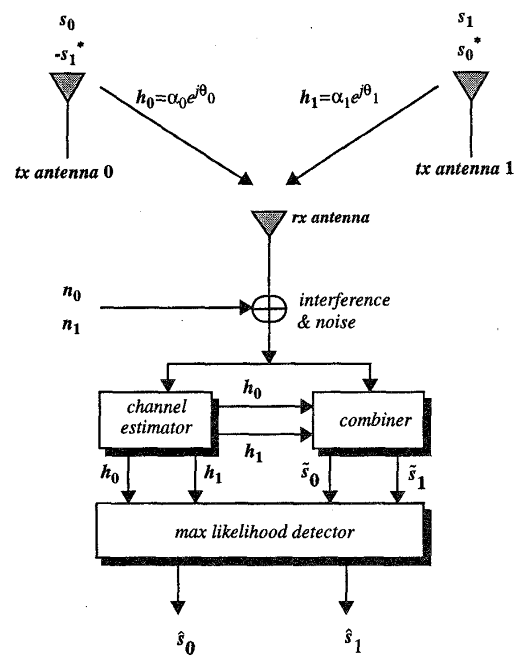

Figure 1 The New Two Branch Transmit Diversity Scheme with 1 Receiver.

2.1 The Encoding and Transmission Sequence

At a given symbol period 2 signals are simultaneously transmitted from the 2 antennas. The signal transmitted from antenna 0 is denoted by s_0, and from antenna 1 by s_1. During the next symbol period signal ($-s_1^*$) is transmitted from antenna 0, and signal s_0^* is transmitted from antenna 1, where * is the complex conjugate operation. This sequence is shown in Table 1.

	antenna 0	antenna 1
time t	s_0	s_1
time $t + T$	$-s_1^*$	s_0^*

Table 1. The Encoding and Transmission Sequence for the New Transmit Diversity Scheme.

In Table 1, the encoding is done in space and time (space-time coding). However, the encoding may also be done in space and frequency. Instead of 2 adjacent symbol periods, 2 adjacent carriers may be used (space-frequency coding).

The channel including the effects of the transmit chain, the airlink, and the receive chain, at time t, may be modeled by a complex multiplicative distortion $h_0(t)$ for transmit antenna 0 and $h_1(t)$ for transmit antenna 1. Assuming that fading is constant across 2 consecutive symbols, we can write:

$$h_0(t) = h_0(t+T) = h_o = \alpha_0 e^{j\theta_0}$$
$$h_1(t) = h_1(t+T) = h_1 = \alpha_1 e^{j\theta_1} \qquad (1)$$

where T is the symbol duration. The received signals can then be expressed as:

$$r_0 = r(t) = h_0 s_0 + h_1 s_1 + n_0$$
$$r_1 = r(t+T) = -h_0 s_1^* + h_1 s_0^* + n_1 \qquad (2)$$

where r_0 and r_1 are the received signals at time t and $t+T$, and n_0 and n_1 are complex random variables representing receiver noise and interference.

2.2 The Combining Scheme

The combiner shown in Figure 1 builds the following 2 combined signals that are sent to the maximum likelihood detector:

$$\tilde{s}_0 = h_0^* r_0 + h_1 r_1^*$$
$$\tilde{s}_1 = h_1^* r_0 - h_0 r_1^* \qquad (3)$$

Substituting Equation 1 and Equation 2 into Equation 3, we get:

$$\tilde{s}_0 = (\alpha_0^2 + \alpha_1^2) s_0 + h_0^* n_0 + h_1 n_1^*$$
$$\tilde{s}_1 = (\alpha_0^2 + \alpha_1^2) s_1 - h_0 n_1^* + h_1^* n_0 \qquad (4)$$

It is important to note that the combining scheme in Equation 3 is different from maximal ratio receiver combining (MRRC). However, the resulting combined signals in Equation 4 are equivalent to that obtained from MRRC with 1 transmit and 2 receive antennas[1]. Therefore, the resulting diversity order from the new transmit diversity scheme with 2 transmit antennas and 1 receiver antenna is equal to that of MRRC with 1 transmit and 2 receive antennas.

The combining scheme in Equation 3 assumes coherent detection which requires the knowledge of the channel at the receiver. New detection schemes for the new transmitter diversity scheme with no channel estimation are described in [9].

2.3 The Maximum Likelihood Decision Rule

The combined signals in Equation 4 are then sent to the maximum likelihood detector. Assuming n_0 and n_1 are Gaussian distributed, the maximum likelihood decision rule at the receiver for these received signals is to choose the signal pair s_0 and s_1 such that the following distance metric is minimized:

$$d^2(r_0, h_0 s_0 + h_1 s_1) + d^2(r_1, -h_0 s_1^* + h_1 s_0^*) \qquad (5)$$

where $d^2(\mathbf{x},\mathbf{y})$ is the squared Euclidean distance between signals $\mathbf{x}$ and $\mathbf{y}$ calculated by the following expression:

$$d^2(x, y) = (x-y)(x^* - y^*) \qquad (6)$$

Expanding Equation 5 and using Equation 3 and Equation 6, we obtain the following decision rule for detecting s_0:

Choose s_i iff:
$$(\alpha_0^2 + \alpha_1^2)|s_i|^2 - \tilde{s}_0 s_i^* - \tilde{s}_0^* s_i \le$$
$$(\alpha_0^2 + \alpha_1^2)|s_k|^2 - \tilde{s}_0 s_k^* - \tilde{s}_0^* s_k \quad \forall i \ne k \qquad (7)$$

or equivalently,:

Choose s_i iff:
$$(\alpha_0^2 + \alpha_1^2 - 1)|s_i|^2 + d^2(\tilde{s}_0, s_i) \le$$
$$(\alpha_0^2 + \alpha_1^2 - 1)|s_k|^2 + d^2(\tilde{s}_0, s_k) \quad \forall i \ne k \qquad (8)$$

For PSK signals (equal energy constellations):

$$|s_i|^2 = |s_k|^2 = E_s \quad \forall i, k \qquad (9)$$

where E_s is the energy of the signal. Therefore, for PSK signals, the decision rule for detecting s_0 in Equation 8 may be simplified to:

Choose s_i iff:
$$d^2(\tilde{s}_0, s_i) \le d^2(\tilde{s}_0, s_k) \quad \forall i \ne k \qquad (10)$$

Similarly, the decision rule for signal s_1 is to choose signal s_i iff:

$$(\alpha_0^2 + \alpha_1^2 - 1)|s_i|^2 + d^2(\tilde{s}_1, s_i) \le$$
$$(\alpha_0^2 + \alpha_1^2 - 1)|s_k|^2 + d^2(\tilde{s}_1, s_k) \quad \forall i \ne k \qquad (11)$$

or for PSK signals:

Choose s_i iff:
$$d^2(\tilde{s}_1, s_i) \le d^2(\tilde{s}_1, s_k) \quad \forall i \ne k \qquad (12)$$

3. System Model

Figure 2 shows a simplified block diagram for the transmitter of the transmit diversity scheme with trellis-coded modulation.

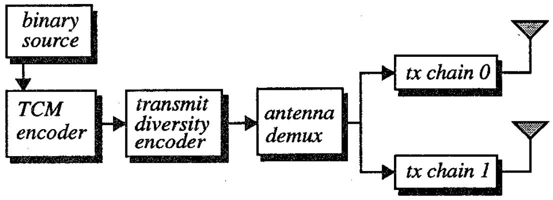

Figure 2 Simplified Transmitter Block Diagram of the Transmit Diversity Scheme with Trellis Coding.

The binary information is fed to a TCM encoder which generates complex numbers representing a constellation symbol. The transmit diversity encoder takes 2 constellation symbols s_0 and s_1 and generates the block s_0, s_1, $-s_1^*$, s_0^*, and transmits them as shown in Table 1 and illustrated in Figure 1. The resulting received baseband signals at time t and $t+T$ are denoted by r_0 and r_1, respectively, and are as represented in Equation 2.

Figure 3 shows a simplified block diagram for the receiver.

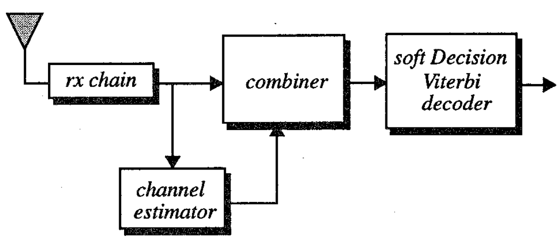

Figure 3 Simplified Block Diagram of the Receiver.

The decoding scheme at the receiver is comprised of 2 functions: a combining scheme and soft decision Viterbi decoding. Using the estimates of the channel, the combiner builds the following 2 combined signals that are sent to the Viterbi decoder:

$$\begin{aligned}\tilde{s}_0 &= \tilde{h}_0^* r_0 + \tilde{h}_1 r_1^* \\ \tilde{s}_1 &= \tilde{h}_1^* r_0 - \tilde{h}_0 r_1^*\end{aligned} \tag{13}$$

The Viterbi decoder builds the following metric for the hypothesized branch symbol s_i corresponding to the first transmitted symbol s_0:

$$M(s_0, s_i) = (\alpha_0^2 + \alpha_1^2 - 1)|s_i|^2 + d^2(\tilde{s}_0, s_i) \tag{14}$$

Similarly, the Viterbi decoder builds the following metric for the hypothesized branch symbol s_i corresponding to the second transmitted symbol s_1:

$$M(s_1, s_i) = (\alpha_0^2 + \alpha_1^2 - 1)|s_i|^2 + d^2(\tilde{s}_1, s_i) \tag{15}$$

If the TCM encoder is a multiple TCM (MTCM) encoder (with 2 hypothesized symbol per branch s_i and s_j): then the Viterbi decoder builds the following metric:

$$M[(s_0, s_1), (s_i, s_j)] = M(s_0, s_i) + M(s_1, s_j) \tag{16}$$

or equivalently:

$$\begin{aligned}&M[(s_0, s_1), (s_i, s_j)] = \\ &d^2(r_0, \tilde{h}_0 s_i + \tilde{h}_1 s_j) + d^2(r_1, \tilde{h}_1 s_i^* - \tilde{h}_0 s_j^*)\end{aligned} \tag{17}$$

4. Performance Analyses in Slow Fading

To keep the analyses simple, we assume that the fade coefficients are constant across an error event of length $2L$. This corresponds to slow fading. Without any loss of generality, we can compute the probability that a maximum likelihood decoder confuses the correct code sequence:

$$\begin{aligned}S = \{&s_0, s_1, -s_1^*, s_0^*, s_2, s_3, -s_3^*, s_2^* \\ &, \ldots, s_{2L-2}, s_{2L-1}, -s_{2L-1}^*, s_{2L-2}^*\}\end{aligned} \tag{18}$$

with the code sequence

$$\begin{aligned}W = \{&w_0, w_1, -w_1^*, w_0^*, w_2, w_3, -w_3^*, w_2^* \\ &, \ldots, w_{2L-2}, w_{2L-1}, -w_{2L-1}^*, w_{2L-2}^*\}\end{aligned} \tag{19}$$

where these code sequences correspond to possible paths in the trellis of the TCM code diverging from one state and merging into another state.

We assume that n_0 and n_1 are complex gaussian random variables with mean 0 and variance N_0, and that the average energy of constellation signals is $E_s/2$; so that the average transmitted energy from the 2 antennas is E_s. The error event probability, conditioned on the fade coefficients h_0 and h_1, is well approximated by the following Chernoff bound [11].

$$\begin{aligned}&P(S \to W | h_0, h_1) \le \\ &e^{\sum_{i=0,2,\ldots 2L-2} \{|(w_i - s_i)h_0 + (w_{i+1} - s_{i+1})h_1|^2 + |(s_{i+1}^* - w_{i+1}^*)h_0 + (w_i^* - s_i^*)h_1|^2\} \frac{-E_s}{8N_0}}\end{aligned} \tag{20}$$

which gives

$$P(S \to W | h_0, h_1) \le e^{\sum_{i=0}^{2L-1} \{|w_i - s_i|^2 (|h_0|^2 + |h_1|^2)\} \frac{-E_s}{8N_0}} \tag{21}$$

Assuming that the absolute values of h_0 and h_1 are Rayleigh distributed (with average power of one) and are independent, we have:

$$P(S \rightarrow W) \leq$$
$$\int_0^{\infty}\int_0^{\infty} P(S \rightarrow W|h_0, h_1) 4|h_0||h_1| e^{-(|h_0|^2+|h_1|^2)} d|h_0| d|h_1| \quad (22)$$

which gives

$$P(S \rightarrow W) \leq \frac{1}{\left(1+\frac{E_s}{8N_0}\sum_{i=0}^{2L-1}|w_i - s_i|^2\right)^2} \quad (23)$$

since the distances between all possible sequences are less than the free distance (d_{free}) of the code, we may then conclude that:

$$P(\text{ error event}) \leq \frac{1}{\left(d_{free}^2 \cdot \frac{E_s}{8N_0}\right)^2} \quad (24)$$

This shows that for the error event probability to be minimized, the trellis code should have maximum free distance. In other words, the design criteria for the trellis code is the maximization of free distance. We may hence conclude that optimal trellis codes designed for AWGN are also optimal for use with the transmit diversity scheme in Rayleigh fading. We may further conclude that the asymptotic coding gain obtained using the trellis codes is comparable with the coding gain in the AWGN channel.

Furthermore, the scheme in [1] has been generalized to a higher number of transmit antennas [12]. The design criteria and the resulting error performance apply to the generalized scheme presented in that paper.

One final remark must be made. Clearly the performance bound established above is for error event probability, and hence the BER is not necessarily optimized.

5. Conclusions

The criteria for the selection of the trellis codes used with the simple transmit diversity scheme proposed in [1] has been established. It has been shown that optimal trellis codes designed for AWGN are also optimal for use with the transmit diversity scheme in slow Rayleigh fading and that the asymptotic coding gain obtained using the trellis codes is comparable with the coding gain in the AWGN channel. We have also shown that the transmit diversity scheme used with trellis codes provides a diversity order of 2 and additional coding gain.

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