1. Fix $n \in \mathbb{N}$ and, for each $0 \leq j \leq n$, consider the polynomial $b_{j,n}(t) := \binom{n}{j} t^j (1-t)^{n-j} \in \mathbb{Q}[t]$.
   (a) Show that the polynomials $b_{0,n}(t), b_{1,n}(t), \ldots, b_{n,n}(t)$ form a basis for $\mathbb{Q}[t]_{\leq n}$.
   (b) Prove that $\sum_{j=0}^{n} b_{j,n}(t) = 1$.

2. For any square matrix $B$ with entries in $\mathbb{K}$, prove that there is a nonzero polynomial $f \in \mathbb{K}[t]$ which has $B$ as a root.

3. Show that the set $\mathfrak{sl}_n(\mathbb{K})$, consisting of all $(n \times n)$-matrices with trace equal to zero, is a linear subspace $\mathfrak{sl}_n(\mathbb{K})$ of $\mathbb{K}^{n \times n}$. Moreover, find a basis for $\mathfrak{sl}_n(\mathbb{K})$ and compute its dimension.