

# Problem Set #20

Due: Thursday, 8 March 2012

1. Compute the limit of the sequence  $(a_n)_{n=1}^{\infty}$  defined by

$$a_n := \frac{n^2}{\sqrt{n^6+1}} + \frac{n^2}{\sqrt{n^6+2}} + \cdots + \frac{n^2}{\sqrt{n^6+n}} = \sum_{k=1}^n \frac{n^2}{\sqrt{n^6+k}}.$$

2. Consider the sequence  $(b_k)_{k=0}^{\infty}$  given by  $b_0 = 2$  and  $b_{k+1} = \frac{1}{3-b_k}$  for  $k \geq 0$ .

(a) Show that this sequence converges.

(b) Calculate  $\lim_{k \rightarrow \infty} b_k$ .

3. (a) Let  $(d_i)_{i=1}^{\infty}$  be a sequence of integers with  $0 \leq d_i \leq 9$ . Prove that series  $\sum_{i=1}^{\infty} (d_i \times 10^{-i})$  converges.

(b) If the sequence  $(d_i)_{i=1}^{\infty}$  is periodic, then show that  $\sum_{i=1}^{\infty} (d_i \times 10^{-i})$  is a rational number. Find this number.