

## Problem Set #6

Due: 17 October 2008

1. A joint density function is given by

$$f(x, y) = \begin{cases} kx^2 & \text{for } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the value of the constant  $k$ .
- (b) Find the probability that  $(x, y)$  satisfies  $x + y \leq 2$ .
- (c) Find the probability that  $(x, y)$  satisfies  $x \leq 1$  and  $y \leq 1/2$ .
2. (a) Find the average value of the sum of the squares of three numbers  $x, y, z$  where each number is between 0 and 2.
- (b) For  $a > 0$  find the volume under the graph of  $z = e^{-(x^2+y^2)}$  above the disk  $x^2 + y^2 \leq a^2$ . What happens to the volume as  $a \rightarrow \infty$ ?
3. Find the total area of the region inside the cardioid  $r = 1 - \cos(\theta)$  and outside the circle  $r = 1$ .