Problem Set #5

Due: 15 October 2010

1. Consider the surface defined by the equation

$$x^3 z + x^2 y^2 + \sin(yz) = -3\,.$$

(a) Find an equation for the plane tangent to this surface at the point $(-1, 0, 3)$.
(b) Parametrize the line normal to this surface at the point $(-1, 0, 3)$.

2. Consider the path $\vec{\boldsymbol{\beta}}\colon (0, \pi) \to \mathbb{R}^2$ given by $\vec{\boldsymbol{\beta}}(t) := \sin(t)\vec{\boldsymbol{\imath}} + \big(\cos(t) + \ln(\tan(t/2))\big)\vec{\boldsymbol{\jmath}}$. The underlying curve is called the ***tractrix***.
(a) Show that derivative $\vec{\boldsymbol{\beta}}'(t)$ is nonzero at everywhere except $t = \pi/2$.
(b) Show that the length of the segment of the tangent of the tractrix between the point of tangency and the y-axis is constantly equal to 1.

Hint. The identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ may be useful in part (a).

3. **(a)** Show that the differentiable path $\vec{\boldsymbol{\gamma}}\colon \mathbb{R} \setminus \{\vec{\boldsymbol{0}}\} \to \mathbb{R}^3$ given by

$$\vec{\boldsymbol{\gamma}}(t) = e^{2t}\vec{\boldsymbol{\imath}} + \ln|t|\vec{\boldsymbol{\jmath}} + \tfrac{1}{t}\vec{\boldsymbol{k}}$$

is a flow line of the vector field $\vec{\boldsymbol{F}}\colon \mathbb{R}^3 \to \mathbb{R}^3$ defined by $\vec{\boldsymbol{F}}(x, y, z) = 2x\vec{\boldsymbol{\imath}} + z\vec{\boldsymbol{\jmath}} - z^2\vec{\boldsymbol{k}}$.

(b) Find the flow lines of the vector field $\vec{\boldsymbol{G}}\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $\vec{\boldsymbol{G}}(x, y) = x\vec{\boldsymbol{\imath}} + 2y\vec{\boldsymbol{\jmath}}$.