Problem Set #2

Due: Friday, January 19, 2007

1. The *transpose* A^{T} of an $(m \times n)$-matrix A is obtained from A by interchanging the rows with the columns; in other words if $A = [a_{i,j}]$ then $A^{\mathsf{T}} = [a_{j,i}]$. A matrix A is *symmetric* if $A^{\mathsf{T}} = A$ and *skew-symmetric* if $A^{\mathsf{T}} = -A$.

(a) Prove that the set W_{skew} of all skew-symmetric $(n \times n)$-matrices is a subspace of $\mathbb{Q}^{n \times n}$.

(b) Let W_{sym} be the subspace of $\mathbb{Q}^{n \times n}$ consisting of all symmetric matrices. Prove that $\mathbb{Q}^{n \times n} = W_{\text{sym}} \oplus W_{\text{skew}}$.

2. Prove or give a counterexample to following: if U_1, U_2 and W are subspaces of V such that $U_1 \oplus W = U_2 \oplus W$ then $U_1 = U_2$.

3. Let $P := \mathbb{R}[t]_{\leq 2}$ the real vector space of all polynomial functions of degree at most 2 and consider $V := \mathbb{R}^P$, the real vector space of all functions from P to $\mathbb{R}$. Determine the linear independence or dependence of the following lists (f_1, f_2, f_3) in V.

(a) for $p \in P$, let $f_1(p) := p(0)$, $f_2(p) := p(1)$ and $f_3(p) := p(2)$;

(b) for $p \in P$, let $f_1(p) := p(0)$, $f_2(p) := \int_0^1 p(t)\, dt$ and $f_3(p) := \int_{-1}^1 p(t)\, dt$.