Problem Set #12
Due: Wednesday, April 4, 2007

1. **(a)** Find the Jordan Canonical Form of the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

   (b) Let $T \in \operatorname{End}(\mathbb{C}^n)$. Suppose that the characteristic polynomial f of T and minimal polynomial g of T satisfy $f(t) = g(t)(t-i)$ and $\big(g(t)\big)^2 = f(t)(t^2+1)$. Determine the Jordan Canonical Form of T.

2. Find a list of complex matrices, as long as possible, such that
   - the characteristic polynomial of each matrix is $(t+3)^4(t+5)^2$,
   - the minimal polynomial of each matrix is $(t+3)^2(t+5)$,
   - no two matrices in the list are similar to each other.

3. Let $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \in \mathbb{C}^{2\times 2}$ and consider $T \in \operatorname{End}(\mathbb{C}^{2\times 2})$ defined by $T(X) = XB - BX$. Find the Jordan Canonical Form for T.