Problem Set #9
MATH 387 : 2015

Due: Thursday, 12 March 2015

1. Consider two perpendicular radii $\overline{OA}$ and $\overline{OB}$ in a circle $\Gamma$ centred at the point $O$. Let $C$ be the midpoint of $\overline{OB}$, and let $\overline{CD}$ be the angle bisector of $\angle ACO$ where $D$ is between $O$ and $A$. If $\overline{DE}$ is perpendicular to the line $\overline{OA}$ and $E$ lies on the circle $\Gamma$, then prove that $\overline{AE}$ is the side of a regular pentagon inscribed in $\Gamma$.

Hint. Use Problem 8.3, to find the length of $\overline{AE}$.

2. In Cartesian plane over the ordered field $\mathbb{k}$, consider an angle $\alpha$ formed by two rays lying on lines of slope $m$ and $m'$. The tangent of $\alpha$ is defined to be

$$\tan(\alpha) = \pm \frac{m' - m}{1 + m \cdot m'},$$

where we take the positive sign if the angle is acute and the negative sign if the angle is obtuse. Using this definition, verify that for any two acute angles $\alpha$ and $\beta$, we have

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha) \cdot \tan(\beta)}.$$

3. In the Cartesian plane over an ordered field $\mathbb{k}$, consider a right triangle $ABC$ where $\angle ABC$ is a right angle. Let $D$ and $E$ be the midpoints of the segments $AB$ and $AC$ respectively. Show that there exists a line segment $\overline{FG}$ such that $F$ is between $B$ and $D$, $G$ is between $C$ and $E$, $\overline{FG}$ is parallel to $BC$, and $\overline{EF}$ is parallel to $BG$ if and only if $\sqrt{2} \in \mathbb{k}$.