Problem Set #11

MATH 387 : 2015

Due: Thursday, 26 March 2015

1. For a proper spherical triangle ABC, prove the following half-angle formulas

$$\sin\left(\tfrac{\alpha}{2}\right) = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin(b)\sin(c)}} \qquad \cos\left(\tfrac{a}{2}\right) = \sqrt{\frac{\cos(\sigma-\beta)\cos(\sigma-\gamma)}{\sin(\beta)\sin(\gamma)}}$$

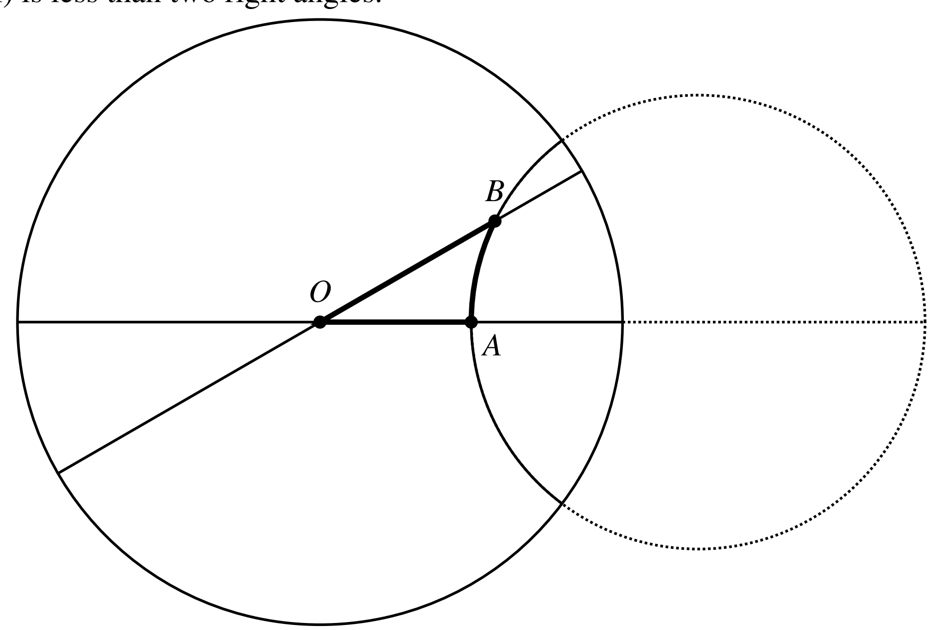
where $s := \frac{1}{2}(a+b+c)$ and $\sigma := \frac{1}{2}(\alpha+\beta+\gamma)$.

Hint. Use the addition formula, to prove the identities

$$2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right) = \cos(\phi)-\cos(\theta) \qquad \sin\left(\tfrac{\theta}{2}\right) = \pm\sqrt{\tfrac{1-\cos(\theta)}{2}}\,.$$

2. A ray Aa is a *limiting parallel* to the ray Bb if either they are coterminal, or if they lie on distinct lines note equal to the line AB, they do not meet, and every ray in the interior of the angle $\angle BAa$ meets the ray Bb. If Aa is a limiting parallel to Bb and the rays Aa and Bb lie on distinct lines, then show that the lines carrying these rays do not meet.

3. Show that the sum of the interior angles for any triangle in the Conformal Disk Model (a.k.a. D-model) is less than two right angles.



Hint. It is enough to show that there is one triangle in the Conformal Disk Model for which the sum of the interior angles is less than two right angles.