Problem Set #12  
MATH 387 : 2015  

Due: Thursday, 2 April 2015

1. Prove that any rigid motion is a product of at most three reflections.

   **Hint.** Any rigid motion is completely determined by the images of any three non-collinear points.

2. The *hyperbolic sine* and *hyperbolic cosine* functions are defined as $\sinh(x) := \frac{1}{2}(e^x - e^{-x})$ and $\cosh(x) := \frac{1}{2}(e^x + e^{-x})$. Given a hyperbolic triangle with vertices $A, B, C$, side lengths $a, b, c$, and interior angles $\alpha, \beta, \gamma$, the *hyperbolic law of cosines* asserts that

   $$\cosh(a) = \cosh(b) \cosh(c) - \sinh(c) \sinh(b) \cos(\alpha).$$

   (a) Verify that $\cosh^2(x) - \sinh^2(x) = 1$.

   (b) Using the hyperbolic law of cosines, derive the *hyperbolic law of sines* which asserts that

   $$\frac{\sinh(a)}{\sin(\alpha)} = \frac{\sinh(b)}{\sin(\beta)} = \frac{\sinh(c)}{\sin(\gamma)}.$$

   **Hint.** Express the fraction $\frac{\sin^2(\alpha)}{\sinh^2(a)}$ as symmetric function in $a, b, c$.

3. What was your favourite result in the course? Provide a short paragraph explaining your answer.