Queen's Algebraic Geometry
— Seminar —

Counting Conics

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Abstract

In the plane, two points are sufficient to determine the line passing through them. It is a fun exercise to check that three points suffice to determine a circle, but how many points are required to determine an ellipse or a hyperbola? What if we replace one of the point conditions by a tangency condition: How many conics pass through a given number of points and are tangent to a given line or a given conic? It is not even clear that these questions are well-posed. The answers may depend on which points, lines and conics we are given. Nineteenth and twentieth century geometers struggled to make sense of these questions, to show that with the proper interpretation they admit clean answers, and to put the subject of enumerative algebraic geometry on a firm mathematical foundation.

Enumerative questions about conics are intrinsically interesting but they also serve as a guide to algebraic geometry. The tools developed to solve these problems reveal deep geometric phenomena. In retrospect many of these seem more valuable than the answers to the original questions. With this in mind, I'll focus on a problem that leads to a lot of interesting geometry. Steiner asked for the number of plane conic curves tangent to five given conics. In solving this problem, we'll meet many interesting objects in algebraic geometry – moduli spaces, Chow rings and blow-ups. I hope to make these objects accessible to the non-expert.

This talk involves joint work with Amy Ksir and Andrew Bashelor.

Monday, October 17, 2004
4:45pm – 5:45pm
319 Jeffery Hall