Queen's Algebraic Geometry
— Seminar —

STANLEY DECOMPOSITIONS AND HILBERT DEPTH IN THE KOSZUL COMPLEX

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Abstract

Stanley decompositions of multigraded modules M over polynomials rings have been discussed intensively in recent years. There is a natural notion of depth that goes with a Stanley decomposition, called the *Stanley depth*. Stanley conjectured that the Stanley depth of a module M is always at least the (classical) depth of M. In this paper we introduce a weaker type of decomposition, which we call *Hilbert decomposition*, since it only depends on the Hilbert function of M, and an analogous notion of depth, called *Hilbert depth*. Since Stanley decompositions are Hilbert decompositions, the latter set upper bounds to the existence of Stanley decompositions. The advantage of Hilbert decompositions is that they are easier to find.

We test our new notion on the syzygy modules of the residue class field of $K[X_1, \ldots, X_n]$ (as usual identified with K). Writing $M(n, k)$ for the k-th syzygy module, we show that the Hilbert depth of $M(n, 1)$ is $\lfloor (n+1)/2 \rfloor$. Furthermore, we show that, for $n > k \geq \lfloor n/2 \rfloor$, the Hilbert depth of $M(n, k)$ is equal to $n - 1$. We conjecture that the same holds for the Stanley depth. For the range $n/2 > k > 1$, it seems impossible to come up with a compact formula for the Hilbert depth. Instead, we provide very precise asymptotic results as n becomes large.

This talk is based on joint work with Chr. Krattenthaler and J. Uliczka.

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16:30 – 17:30
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