Queen's Algebraic Geometry
— Seminar —

AUTOMORPHIC CUSP FORMS AND CUSPIDAL SINGULARITIES

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Abstract

The purpose of this talk is to show that cusp forms and cusp singularities are not merely homonyms but are related on a deeper level. More specifically, we'll discuss the relation between Fuchsian triangle groups and their automorphic forms, and quasi-homogeneous singularities of affine plane curves. The prototypical example is the modular group PSL(2,Z): the quotient of the nonzero tangent bundle on the upper-half plane by the action of PSL(2,Z) is biholomorphic to the complement of the plane curve $z^3 - 27w^2 = 0$. This can be shown using the fact that the algebra of modular forms is doubly generated and $\Delta = g_2^3 - 27g_3^2$ is the cusp form of minimal degree. This result is generalized to include all (p, q, ∞)-triangle groups and, respectively, curves of the form $z^q + w^p = 0$, for p, q coprime. An interesting biproduct is a geometric structure on the complement of a (p,q)-torus knot. The proof uses ideas of Milnor and Dolgachev, which they introduced in their studies of the spectra of the algebras of automorphic forms of cocompact triangle groups (and, more generally, uniform lattices). It turns out that the same approach, with some modifications, allows to handle the cuspidal case.

Monday 7 March 2011
16:30 – 17:30
319 Jeffery Hall