Joachim Mahnkopf, Jerusalem
Values of twisted automorphic L-Functions

Let π be a cuspidal automorphic representation of $GL_n(\mathbb{A})$, $\mathbb{A}$ the ring of Adeles of $\mathbb{Q}$ and fix a prime $p \in \mathbb{N}$. We assume that π_p is unramified. Let furthermore χ be an idele class character of finite order with infinity component $\chi_\infty = \mathrm{id}$ and conductor $f_\chi = p^e$ a p-power. We prove a formula for the (twisted) values of the automorphic L-Function $L(\pi \otimes \chi, s)$, which can be expressed using (de-Rham) cohomology classes of the symmetric space of $GL_{n-1}(\mathbb{R})$. The proof exploites the decomposition of the Rankin-Selberg convolution on $GL_n \times GL_{n-1}$

$$L(\pi \times \sigma(\chi), s) = L(\pi \otimes \chi, s + k_1) \prod_{i>2} L(\pi, s + k_i)$$

where σ is the induced representation $\mathrm{Ind}(\chi||^{k_1}, ||^{k_2}, \dots, ||^{k_{n-1}})$, $k_i \in \mathbb{R}$ and we thus may use use the zeta integral of $L(\pi \times \sigma(\chi), s)$ to derive an integral formula for the values $L(\pi\otimes\chi, s+k_1)$. For the group GL_2 this coincides with a formula of A. Weil for the twist of modular forms. Assuming that π has non-trivial cohomology we derive in the case GL_3 the algebraicity of $L(\pi \otimes \chi, 1)/\Omega(\pi)$ for a certain period $\Omega(\pi) \in \mathbb{C}^*$ and also construct a distribution μ on $\mathbb{Z}_p^*$ which interpolates the L-Function $\int_{\mathbb{Z}_p^*} \chi_p \, d\mu = L(\pi \otimes \chi, 1)/\Omega(\pi)$.

Ernst Kani, Kingston
Diagonal Quotient Surfaces and a Question of Mazur

Let E be an elliptic curve over a number field K, N a *prime* and

$$\bar{\rho}_{E,N} = \bar{\rho}_{E/K,N} : G_K = Gal(\bar{K}/K) \to Aut(E[N]) \simeq Gl_2(\mathbb{Z}/N\mathbb{Z})$$

its associated Galois representation mod N. A question of Mazur (1978) may be generalized as follows.
Question: To what extent is (the isogeny class of) E/K determined by (the isomorphism class of) $\bar{\rho}_{E,N}$?

This question was studied by Kraus, Oesterlé, Halberstadt and others. Frey proposed in 1988
Conjecture 0: $\exists M = M(E,K)$ such that the set

$$S_{N,E}(K) := \{E'/K \;:\; E' \not\sim E,\; \bar{\rho}_{E,N} \cong \bar{\rho}_{E',N}\} = \emptyset, \quad \forall\, N \geq M.$$

Using the results of Wiles, he recently showed that this conjecture is (for $K = \mathbb{Q}$) equivalent to the *Asymptotic Fermat Conjecture.*

In 1994 H. Darmon formulated several conjectures which (partially) generalize Conjecture 0. In this talk the following stronger and more precise version of one of his conjectures was presented:

Conjecture 1: For every (prime) $N \geq 23$, the set

$$\mathcal{S}_N^*(K) := \{(E, E')_{/K} : \exists \text{ non-trivial } G_K\text{-isometry } \Psi : E[N] \xrightarrow{\sim} E'[N]\}$$

is finite. (Here: a G_K-isometry Ψ is called *trivial* if there exists a cyclic isogeny $f : E \to E'$ of degree $d \leq 27$, $d \neq 22, 23, 26$, such that $\Psi = k \cdot f|_{E[N]}$ for some $k \in \mathbb{Z}$.)

This conjecture has a nice interpretation in terms of the *Modular Diagonal Quotient Surfaces*

$$Z_{N,E} := \Delta_\varepsilon \backslash (X(N) \times X(N))$$

where $X(N)$ is the usual modular curve of level N,

$$\Delta_\varepsilon = \{(g, \alpha_\varepsilon(g)) : g \in G_N\} \leq G_N \times G_N, \; G_N = \mathrm{Sl}_2(\mathbb{Z}/N\mathbb{Z})/\pm 1$$

$$\alpha_\varepsilon : g \mapsto Q_\varepsilon \, g \, Q_\varepsilon^{-1}, \; \forall g \in G_N; \; Q_\varepsilon = \begin{pmatrix} \varepsilon & 0 \\ 0 & 1 \end{pmatrix}, \; \varepsilon \in (\mathbb{Z}/N\mathbb{Z})^\times.$$

Conjecture 2: For $N \geq 23$, every curve $C \subset Z_{N,\varepsilon}$ of genus ≤ 1 is a Hecke curve: $C = \bar{T}_{n,k}$ (induced by a Hecke correspondence).

It is not difficult to see that Conjecture 1 $\Rightarrow$ Conjecture 2. The converse is also true if one uses *Lang's Conjecture* (about rational points on surfaces of general type) because one has:

Theorem 1 (C.F. Hermann, E. Kani, W. Schanz): $Z_{N,\varepsilon}$ is of general type if $N \geq 13$.

Finally, some partial evidence for Conjecture 2 was discussed, such as its relation to an analogue of the *Minimality Conjecture* of Hirzebruch for Hilbert Modular Surfaces.

Bernhard Köck, Karlsruhe
Operations on Locally Free Classgroups

Let K be a number field and Γ a finite group. In order to construct annihilators of Stickelberger type for the locally free classgroup $Cl(\mathcal{O}_K\Gamma)$, Cassou-Noguès and Taylor have shown in 1985 that the Adams operations on the classical ring of virtual characters of Γ induce certain endomorphisms $\psi_K, k \geq 1$, on $Cl(\mathcal{O}_K\Gamma)$ in Fröhlich's Hom-description.

Recently, Burns and Chinburg have studied the question whether there is an algebraic description of ${\psi_K}^{CNT}$, for instance in terms of power operations on $\mathcal{O}_K\Gamma$-modules, and they have established a formula for $\psi_K(x)$ for certain elements x in $Cl(\mathcal{O}_K\Gamma)$ coming from a tame Galois extension of K.

We showed that ψ_k is a simply definable symmetric power operation on $Cl(\mathcal{O}_K\Gamma)$. For the proof of this result we use topological arguments based on the construction of power operations on higher K-theory. As an application of