

# Endomorphisms of Jacobians of Modular Curves

Ernst Kani

Department of Mathematics and Statistics  
Queen's University, Kingston, Ontario, Canada, K7L 3N6  
Kani@mast.queensu.ca

**Abstract.** Let  $X_\Gamma = \Gamma \backslash \mathfrak{H}^*$  be the modular curve associated to a congruence subgroup  $\Gamma$  of level  $N$  with  $\Gamma_1(N) \leq \Gamma \leq \Gamma_0(N)$ , and let  $X = X_{\Gamma, \mathbb{Q}}$  be its canonical model over  $\mathbb{Q}$ . The main aim of this paper is to show that the endomorphism algebra  $\text{End}_{\mathbb{Q}}^0(J_X)$  of its Jacobian  $J_X/\mathbb{Q}$  is generated by the Hecke operators  $T_p$ , with  $p \nmid N$ , together with the “degeneracy operators”  $D_{M,d}, D_{M,d}^t$ , for  $dM|N$ . This uses the fundamental results of Ribet on the structure of  $\text{End}_{\mathbb{Q}}^0(J_X)$  together with a basic result on the classification of the irreducible modules of the algebra generated by these operators.

## AMS Subject Classification (2000)

11G18 Arithmetic aspects of modular and Shimura varieties  
11F32 Modular correspondences

The following are also relevant:

11F11 Modular forms, one variable  
14G35 Arithmetic problems: Modular and Shimura varieties

**Key Words:** Modular curves, modular forms, Jacobians, endomorphism algebras, Hecke algebras, Hecke correspondences, degeneracy operators, Atkin-Lehner theory, Shimura construction