Jacobians isomorphic to a product of two elliptic curves and ternary quadratic forms

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Abstract: The purpose of this paper is to study the question of whether or not the product surface $E_1 \times E_2$ can be the Jacobian of a smooth curve of genus 2, particularly when E_1 and E_2 are elliptic curves with complex multiplication. This question was first raised Hayashida and Nishi in 1965.

By using the refined Humbert invariant, this question can be translated into an interesting classification problem about quadratic and ternary forms. This leads to a complete solution of the above question: there are precisely 15 isomorphism classes of such product surfaces which cannot be Jacobians.