

# The number of genus 2 covers of an elliptic curve

Ernst Kani

**Abstract.** The main aim of this paper is to determine the number  $c_{N,D}$  of genus 2 covers of an elliptic curve  $E$  of fixed degree  $N \geq 1$  and fixed discriminant divisor  $D \in \text{Div}(E)$ . In the case that  $D$  is reduced, this formula is due to Dijkgraaf.

The basic technique here for determining  $c_{N,D}$  is to exploit the geometry of a certain compactification  $\mathcal{C} = \mathcal{C}_{E,N}$  of the universal genus 2 curve over the *Hurwitz space*  $H_{E,N}$  which classifies (normalized) genus 2 covers of degree  $N$  of  $E$ . Thus, a secondary aim of this paper is to study the geometry of  $\mathcal{C}$ . For example, the structure of its degenerate fibres is determined, and this yields formulae for the numerical invariants of  $\mathcal{C}$  which are also of independent interest.