Binary Theta Series and Modular Forms with Complex Multiplication

Abstract:

The main purpose of this paper is to give an intrinsic interpretation of the space $\Theta(D)$ generated by the binary theta series $\vartheta_f$ attached to the positive binary quadratic forms $f$ whose discriminant has the form $D(f) = D/t^2$, for some integer $t$. It turns out that $\Theta(D) = M_1^{CM}(|D|, \psi_D)$, the space of modular forms of weight 1 and of level $|D|$ which have complex multiplication (CM) by their Nebentypus character $\psi_D = (D)$; cf. Theorem 1. As an application, we obtain a structure theorem (Theorem 16) of the space $M_1^{CM}(|D|, \psi_D)$.

The proof of this theorem rests on the results of [8] together with a characterization of the newforms $f$ which have CM by their Nebentypus character in terms of properties of the associated Deligne/Serre Galois representation $\rho_f$; cf. Theorem 9.

Keywords:

Binary quadratic forms, theta series, modular forms, Hecke algebra, newforms, complex multiplication, Galois representations, dihedral groups.

2010 AMS Subject Classification:

11F27 Number Theory: Discontinuous groups and automorphic forms: Theta series; Weil representation; Theta correspondence

11F80 Number Theory: Discontinuous groups and automorphic forms: Galois representations

11E45 Number Theory: Forms and Linear Algebraic Groups: Analytic Theory (Epstein zeta function; relation with automorphic forms and functions)