Invariants of the Surface $\tilde{Z}_{13,1}$

Basic Numerical Invariants:

Geometric:

p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
4	49	57	-39	59	1

Other:

m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	$\mathbb{L}_\infty$	$\mathbb{L}$	$2\mathbb{S}_\infty$	$2\mathbb{S}$
1092	50	6	24	4	16	2	6	2	16	27	39	0	0

The Singularities of the associated singular surface $Z_{13,1}$

The Singularities above P0:

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	$[0,12]$	$+$	1	1	$[13,12],[1,1]$	$[313,650,338]$	$[1,0,169]$
2	$[2,7]$	$+$	2	2	$[2,7],[-1,-3]$	$[53,-598,1690]$	$[10,2,17]$
3	$[6,11]$	$+$	2	2	$[6,11],[1,2]$	$[157,728,845]$	$[10,-2,17]$
4	$[0,8]$	$-$	3	3	$[13,8],[-5,-3]$	$[233,-2314,5746]$	$[1,0,169]$
5	$[3,9]$	$-$	3	3	$[3,22],[-1,-7]$	$[493,-4082,8450]$	$[10,-2,17]$
6	$[4,10]$	$-$	3	3	$[4,23],[1,6]$	$[545,3692,6253]$	$[10,2,17]$

The CM-Singularities above P1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
7	$[0,1]$	$+$	1	1	$[0,1],[-1,0]$	$[1,-13,169]$	$[1,1,127]$
8	$[3,3]$	$+$	1	2	$[3,16],[-1,-5]$	$[313,-2561,5239]$	$[3,3,43]$

The anti-CM-Singularities above P1 (those of type $(-2,-2)$)

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
9	$[0,5]$	$-$	2	3	$[13,5],[5,2]$	$[259,2613,6591]$	$[1,1,127]$
10	$[2,2]$	$-$	2	3	$[2,15],[-1,-7]$	$[259,-3159,9633]$	$[3,3,43]$

The Singularities above P_∞:

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
11	$[1,0]$	1	1	$[13,1]$	1	$[13]$
12	$[2,0]$	1	2	$[13,10]$	4	$[2,2,2,4]$
13	$[3,0]$	1	3	$[13,3]$	3	$[5,2,2]$
14	$[4,0]$	1	4	$[13,9]$	3	$[2,2,5]$
15	$[5,0]$	1	5	$[13,12]$	12	$[2,2,2,2,2,2,2,2,2,2,2,2]$
16	$[6,0]$	1	6	$[13,4]$	4	$[4,2,2,2]$

The Basic Curves on $\tilde{Z}_{13,1}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	24	24	0	-3
8	24	24	0	-3
9	16	16	0	-2
16	16	16	0	-2
17	2	2	0	-3
45	2	2	0	-3

The intersection matrix for the non-exceptional curves:

No	1	8	9	16	17	45
1	−3	270	0	182	0	42
8	270	−3	182	0	42	0
9	0	182	−2	120	0	28
16	182	0	120	−2	28	0
17	0	42	0	28	−3	6
45	42	0	28	0	6	−3

The intersection matrix for the P_0-curves (curves 1...8)

No	1	2	3	4	5	6	7	8
1	−3	1	1	1	1	1	1	270
2	1	−2	0	0	0	0	0	1
3	1	0	−2	0	0	0	0	1
4	1	0	0	−2	0	0	0	1
5	1	0	0	0	−2	0	0	1
6	1	0	0	0	0	−2	0	1
7	1	0	0	0	0	0	−2	1
8	270	1	1	1	1	1	1	−3

The intersection matrix for the P_1-curves (curves 9...16)

No	9	10	11	12	13	14	15	16
9	−2	1	1	1	0	1	0	120
10	1	−3	0	0	0	0	0	1
11	1	0	−3	0	0	0	0	1
12	1	0	0	−2	1	0	0	0
13	0	0	0	1	−2	0	0	1
14	1	0	0	0	0	−2	1	0
15	0	0	0	0	0	1	−2	1
16	120	1	1	0	1	0	1	−2

The intersection matrix for the P_∞-curves (curves 17...45)

No	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
17	−3	1	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0
18	1	−13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1	0	−2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	1	−2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	1	−2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	1	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	1	0	0	0	0	0	−5	1	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	1	−2	1	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	1	−2	0	0	0	0	0	0	0	0	0	0	0
26	1	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	1	−2	1	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	1	−5	0	0	0	0	0	0	0	0
29	1	0	0	0	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	1	−2	1	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−2	1	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−2	1	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−2	1	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−2	1	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−2	1
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−2
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	6	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0

No	37	38	39	40	41	42	43	44	45
17	0	0	0	0	1	0	0	0	6
18	0	0	0	0	0	0	0	0	1
19	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	1
23	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	1
26	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	1
29	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0
36	1	0	0	0	0	0	0	0	0
37	−2	1	0	0	0	0	0	0	0
38	1	−2	1	0	0	0	0	0	0
39	0	1	−2	1	0	0	0	0	0
40	0	0	1	−2	0	0	0	0	1
41	0	0	0	0	−4	1	0	0	0
42	0	0	0	0	1	−2	1	0	0
43	0	0	0	0	0	1	−2	1	0
44	0	0	0	0	0	0	1	−2	1
45	0	0	0	1	0	0	0	1	−3

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{13,1}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
46	1	1	1	0	0	0	−1
47	3	3	4	0	0	0	−1
48	4	6	6	0	0	0	−1
49	9	4	12	0	0	0	−2
50	10	2	18	0	0	0	−4
51	12	5	24	0	0	0	−4
52	14	1	24	1	1	0	−2
53	16	3	24	0	0	0	−2
54	17	6	18	1	1	0	−2
55	22	4	36	2	2	0	−4
56	23	2	24	2	2	0	−2
57	25	5	30	0	0	0	−4
58	27	1	36	1	1	0	−2
59	29	3	30	2	2	0	−2
60	30	6	72	3	3	0	−2

Their intersection numbers with other curves:

a) Those with the curves over P_0:

No	n	k	deg	1	2	3	4	5	6	7	8
46	1	1	1	0	1	0	0	0	0	0	0
47	3	3	4	2	0	0	0	0	0	0	2
48	4	6	6	3	0	0	0	0	0	0	3
49	9	4	12	6	0	0	0	0	0	0	6
50	10	2	18	8	0	1	1	0	0	0	8
51	12	5	24	12	0	0	0	0	0	0	12
52	14	1	24	12	0	0	0	0	0	0	12
53	16	3	24	12	0	0	0	0	0	0	12
54	17	6	18	8	0	1	1	0	0	0	8
55	22	4	36	18	0	0	0	0	0	0	18
56	23	2	24	12	0	0	0	0	0	0	12
57	25	5	30	14	0	1	1	0	0	0	14
58	27	1	36	18	0	0	0	0	0	0	18
59	29	3	30	14	0	1	1	0	0	0	14
60	30	6	72	36	0	0	0	0	0	0	36

b) Those with the curves over P_1:

No	n	k	deg	9	10	11	12	13	14	15	16
46	1	1	1	0	1	0	0	0	0	0	0
47	3	3	4	1	0	1	0	0	0	0	1
48	4	6	6	2	0	0	0	0	0	0	2
49	9	4	12	4	0	0	0	0	0	0	4
50	10	2	18	6	0	0	0	0	0	0	6
51	12	5	24	8	0	0	0	0	0	0	8
52	14	1	24	8	0	0	0	0	0	0	8
53	16	3	24	8	0	0	0	0	0	0	8
54	17	6	18	6	0	0	0	0	0	0	6
55	22	4	36	12	0	0	0	0	0	0	12
56	23	2	24	8	0	0	0	0	0	0	8
57	25	5	30	10	0	0	0	0	0	0	10
58	27	1	36	12	0	0	0	0	0	0	12
59	29	3	30	10	0	0	0	0	0	0	10
60	30	6	72	24	0	0	0	0	0	0	24

c) Those with the curves over P_∞:

No	n	k	deg	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
46	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	3	3	4	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0
48	4	6	6	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	9	4	12	0	2	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
50	10	2	18	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
51	12	5	24	0	0	0	0	1	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0
52	14	1	24	1	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	16	3	24	1	2	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0
54	17	6	18	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	22	4	36	1	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0
56	23	2	24	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	25	5	30	1	4	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
58	27	1	36	2	2	0	0	0	0	2	0	0	0	0	2	0	0	0	0	0	0	0	0
59	29	3	30	2	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0
60	30	6	72	3	4	0	0	0	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0

No	n	k	deg	37	38	39	40	41	42	43	44	45
46	1	1	1	0	0	0	0	0	0	0	0	0
47	3	3	4	0	0	0	0	0	0	0	0	0
48	4	6	6	0	0	0	0	1	0	0	0	0
49	9	4	12	0	0	0	0	0	0	0	0	0
50	10	2	18	0	0	0	0	0	0	0	1	0
51	12	5	24	0	0	0	1	0	1	0	0	0
52	14	1	24	0	0	0	0	0	0	1	0	1
53	16	3	24	0	0	0	0	1	0	0	0	1
54	17	6	18	0	0	0	0	1	0	0	0	1
55	22	4	36	0	0	1	0	0	0	0	0	1
56	23	2	24	0	0	0	0	0	0	0	1	1
57	25	5	30	0	0	0	1	0	0	0	0	1
58	27	1	36	0	0	0	0	0	0	0	0	2
59	29	3	30	0	0	0	0	0	0	0	0	2
60	30	6	72	0	1	0	0	1	0	0	0	3

d) Those of the Hecke curves with each other:

No	n	k	deg	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
46	1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	3	3	4	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0
48	4	6	6	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0
49	9	4	12	0	0	0	−2	0	0	0	0	0	0	0	0	0	0	0
50	10	2	18	0	0	0	0	−4	0	0	0	0	0	0	0	0	0	0
51	12	5	24	0	0	0	0	0	−4	0	0	0	0	0	0	0	0	0
52	14	1	24	0	0	0	0	0	0	−2	0	0	2	0	0	2	2	2
53	16	3	24	0	0	0	0	0	0	0	−2	0	2	2	0	0	2	0
54	17	6	18	0	0	0	0	0	0	0	0	−2	2	0	0	2	0	2
55	22	4	36	0	0	0	0	0	0	2	2	2	−4	2	0	2	2	0
56	23	2	24	0	0	0	0	0	0	0	2	0	2	−2	2	2	0	4
57	25	5	30	0	0	0	0	0	0	0	0	0	0	2	−4	2	2	0
58	27	1	36	0	0	0	0	0	0	2	0	2	2	2	2	−2	0	2
59	29	3	30	0	0	0	0	0	0	2	2	0	2	0	2	0	−2	4
60	30	6	72	0	0	0	0	0	0	2	0	2	0	4	0	2	4	−2