

Invariants of the Surface $\tilde{Z}_{15,1}$

Basic Numerical Invariants:

Geometric:	p_g	$h^{1,1}$	b_2	sgn	c_2	K^2
	4	56	64	-46	66	-6

Other:	m	g	r_0	g_0	r_1	g_1	s_{11}	r_∞	g_∞	h	$\mathbb{L}_\infty$	$\mathbb{L}$	$2\mathbb{S}_\infty$	$2\mathbb{S}$
	1440	73	8	35	6	23	6	12	1	28	20	34	5	6

The Singularities of the associated singular surface $Z_{15,1}$

The Singularities above P0:

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
1	[0, 11]	+	1	1	[15, 11], [4, 3]	[346, 2790, 5625]	[1, 0, 225]
2	[0, 14]	+	1	2	[15, 14], [1, 1]	[421, 870, 450]	[1, 0, 225]
3	[5, 9]	+	2	3	[5, 9], [1, 2]	[106, 690, 1125]	[9, 0, 25]
4	[6, 10]	+	2	3	[6, 25], [-1, -4]	[661, -3180, 3825]	[9, 0, 25]
5	[2, 10]	-	4	4	[2, 25], [-1, -12]	[629, -9060, 32625]	[9, 6, 26]
6	[5, 8]	-	4	4	[5, 8], [-2, -3]	[89, -1020, 2925]	[9, 6, 26]
7	[5, 13]	-	4	4	[5, 13], [-2, -5]	[194, -2250, 6525]	[9, -6, 26]
8	[7, 10]	-	4	4	[7, 10], [2, 3]	[149, 1320, 2925]	[9, -6, 26]

The CM-Singularities above P1 (those of type (-3))

No	Name	Sign	Deg	Orbit	Basis of M_P	Quadratic Form	Reduced Form
9	[0, 1]	+	1	1	[0, 1], [-1, 0]	[1, -15, 225]	[1, 1, 169]
10	[0, 4]	+	1	2	[15, 4], [-4, -1]	[301, -2385, 4725]	[1, 1, 169]
11	[1, 5]	+	2	3	[1, 5], [0, 1]	[31, 165, 225]	[9, 3, 19]
12	[4, 5]	+	2	4	[4, 5], [-1, -1]	[61, -405, 675]	[9, 3, 19]
13	[5, 1]	+	2	3	[5, 1], [-1, 0]	[31, -165, 225]	[9, -3, 19]
14	[5, 4]	+	2	4	[5, 4], [1, 1]	[61, 405, 675]	[9, -3, 19]

The anti-CM-Singularities above P1 (those of type $(-2, -2)$)

– there are none of this type

The Singularities above P_∞ :

No	Name	Degree	Orbit	Type	Length	Continued Fraction Expansion
15	[1, 0]	1	1	[15, 1]	1	[15]
16	[1, 3]	4	2	[3, 1]	1	[3]
17	[1, 5]	2	3	[5, 1]	1	[5]
18	[1, 6]	4	2	[3, 1]	1	[3]
19	[1, 9]	4	2	[3, 1]	1	[3]
20	[1, 10]	2	3	[5, 1]	1	[5]
21	[1, 12]	4	2	[3, 1]	1	[3]
22	[2, 0]	1	4	[15, 4]	2	[4, 4]
23	[2, 5]	2	5	[5, 4]	4	[2, 2, 2, 2]
24	[2, 10]	2	5	[5, 4]	4	[2, 2, 2, 2]
25	[4, 0]	1	6	[15, 1]	1	[15]
26	[7, 0]	1	7	[15, 4]	2	[4, 4]

The Basic Curves on $\tilde{Z}_{15,1}$:

Table of the non-exceptional basic curves

No	p_a	g	δ_C	C^2
1	35	35	0	-4
10	35	35	0	-4
11	23	23	0	-2
18	23	23	0	-2
19	1	1	0	-4
40	1	1	0	-4

The intersection matrix for the non-exceptional curves:

No	1	10	11	18	19	40
1	-4	356	0	240	0	48
10	356	-4	240	0	48	0
11	0	240	-2	158	0	32
18	240	0	158	-2	32	0
19	0	48	0	32	-4	4
40	48	0	32	0	4	-4

The intersection matrix for the P_0 -curves (curves 1...10)

No	1	2	3	4	5	6	7	8	9	10
1	-4	1	1	1	1	1	1	1	1	356
2	1	-2	0	0	0	0	0	0	0	1
3	1	0	-2	0	0	0	0	0	0	1
4	1	0	0	-2	0	0	0	0	0	1
5	1	0	0	0	-2	0	0	0	0	1
6	1	0	0	0	0	-2	0	0	0	1
7	1	0	0	0	0	0	-2	0	0	1
8	1	0	0	0	0	0	0	-2	0	1
9	1	0	0	0	0	0	0	0	-2	1
10	356	1	1	1	1	1	1	1	1	-4

The intersection matrix for the P_1 -curves (curves 11...18)

No	11	12	13	14	15	16	17	18
11	-2	1	1	1	1	1	1	158
12	1	-3	0	0	0	0	0	1
13	1	0	-3	0	0	0	0	1
14	1	0	0	-3	0	0	0	1
15	1	0	0	0	-3	0	0	1
16	1	0	0	0	0	-3	0	1
17	1	0	0	0	0	0	-3	1
18	158	1	1	1	1	1	1	-2

The intersection matrix for the P_∞ -curves (curves 19...40)

No	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
19	-4	1	1	1	1	1	1	1	1	0	1	0	0	0	1	0	0	0	1	1	0	4
20	1	-15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
21	1	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
22	1	0	0	-5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
23	1	0	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
24	1	0	0	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
25	1	0	0	0	0	0	-5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
26	1	0	0	0	0	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	1
27	1	0	0	0	0	0	0	0	-4	1	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	1	-4	0	0	0	0	0	0	0	0	0	0	0	1
29	1	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	1
33	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	1
37	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-15	0	0	1
38	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	1	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-4	1
40	4	1	1	1	1	1	1	1	0	1	0	0	0	1	0	0	0	1	1	0	1	-4

The Hecke curves $T = T_{n,k}$ on $\tilde{Z}_{15,1}$ for $n \leq 30$

Their basic properties:

No	n	k	deg	p_a	g_T	δ	T^2
41	1	1	1	0	0	0	-1
42	1	4	1	0	0	0	-1
43	4	2	6	0	0	0	-1
44	4	7	6	0	0	0	-1
45	16	1	24	0	0	0	-2
46	16	4	24	0	0	0	-2
47	19	2	20	1	1	0	-2
48	19	7	20	1	1	0	-2

Their intersection numbers with other curves:

a) Those with the curves over P_0 :

No	n	k	deg	1	2	3	4	5	6	7	8	9	10
41	1	1	1	0	0	1	0	0	0	0	0	0	0
42	1	4	1	0	1	0	0	0	0	0	0	0	0
43	4	2	6	3	0	0	0	0	0	0	0	0	3
44	4	7	6	3	0	0	0	0	0	0	0	0	3
45	16	1	24	12	0	0	0	0	0	0	0	0	12
46	16	4	24	12	0	0	0	0	0	0	0	0	12
47	19	2	20	10	0	0	0	0	0	0	0	0	10
48	19	7	20	10	0	0	0	0	0	0	0	0	10

b) Those with the curves over P_1 :

No	n	k	deg	11	12	13	14	15	16	17	18
41	1	1	1	0	1	0	0	0	0	0	0
42	1	4	1	0	0	1	0	0	0	0	0
43	4	2	6	2	0	0	0	0	0	0	2
44	4	7	6	2	0	0	0	0	0	0	2
45	16	1	24	8	0	0	0	0	0	0	8
46	16	4	24	8	0	0	0	0	0	0	8
47	19	2	20	6	0	0	0	1	0	1	6
48	19	7	20	6	0	0	1	0	1	0	6

c) Those with the curves over P_∞ :

No	n	k	deg	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
41	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	1	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
43	4	2	6	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
44	4	7	6	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
45	16	1	24	1	2	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	2	0
46	16	4	24	1	2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	2	1
47	19	2	20	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
48	19	7	20	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1

d) Those of the Hecke curves with each other:

No	n	k	deg	41	42	43	44	45	46	47	48
41	1	1	1	-1	0	0	0	0	0	0	0
42	1	4	1	0	-1	0	0	0	0	0	0
43	4	2	6	0	0	-1	0	0	0	0	0
44	4	7	6	0	0	0	-1	0	0	0	0
45	16	1	24	0	0	0	0	-2	0	0	0
46	16	4	24	0	0	0	0	0	-2	0	0
47	19	2	20	0	0	0	0	0	0	-2	0
48	19	7	20	0	0	0	0	0	0	0	-2