Today’s main lesson is:

• to learn a few methods to find the winner of an election, together with two fairness criteria for elections.

**Methods to find the winner of an election**

We will assume that voters pick candidates in order of preference in our elections.

○ **The plurality method:** Under the plurality method, the candidate with the most first-place votes is the winner of the election.

○ **The Borda count method:** Suppose that there are $N$ candidates for an election. For each ballot, we give $N$ points for the first place choice, $N - 1$ points for second place, and so on, with 1 point for last place. We calculate the total points for each candidate according to the points assigned on the ballots. The candidate with the highest total points is the winner of the election.

○ **The plurality-with-elimination method:** Under the plurality-with-elimination method,

  • **Step 1:** Count the first-place votes for each candidate (as we would in the plurality method). If a candidate has a majority (more than half) of the first-place votes, the candidate is the winner. Otherwise, eliminate the candidate(s) with the fewest first-place votes.

  • **Step 2:** Cross out the name(s) of the eliminated candidate(s) from each ballot, and recount the first-place votes. If a candidate has a majority of the first-place votes, the candidate is the winner. Otherwise, eliminate the candidate(s) with the fewest first-place votes.

  • **Step 3, 4 etc:** Repeat the process (each time eliminating one or more candidates from the ballots) until there is a candidate with a majority of the first-place votes. The candidate is the winner of the election.

NOTE: You will notice that as you eliminate candidate(s) in the plurality-with-elimination method, candidates below the eliminated candidate(s) will be shifted upwards in preference on the ballots. That is, if you cross out the candidates in first place on some ballots, the second-place candidate on these ballots will now be in first-place. Since many voters may have chosen a particular candidate for, say, second place, there is a chance for a candidate to rise from low rankings in Step 1 to winning the election in later steps.
Two fairness criteria for elections

- **The majority criterion**: If candidate $X$ has a majority of the first-place votes, then candidate $X$ should be the winner of the election. The candidate with a majority of the first-place votes is called the **majority candidate**.

- **The Condorcet criterion**: If candidate $X$ is preferred by the voters over each of the other candidates in a head-to-head comparison, then candidate $X$ should be the winner of the election. The candidate preferred by the voters over each of the other candidates in a head-to-head comparison is called the **Condorcet candidate**.

**Weekly Assignment 1 (Due: January 16th, 2008)**

You may hand in your complete assignment at the next class (January 15th), or the following day at the math department office (Jeffery 310). Assignments should be stapled and clearly labeled with your full name, student number and the class number.

1. Question 3 of Chapter 1 (Question 3 (a),(b),(c),(f) in the 5th edition textbook)
2. Question 4 of Chapter 1 (Question 4 (a),(b),(d),(e); 5th edition)
3. Question 11 of Chapter 1 (Question 9; 5th edition)
4. Question 12 of Chapter 1 (Question 10; 5th edition)
5. Question 18 (a),(b) of Chapter 1 (Question 18; 5th edition)
   NOTE: If you are using the 5th edition, in (b) of Question 18, change Cleo into Dionne.
6. Question 20 of Chapter 1 (Question 20; 5th edition)
7. Question 28 of Chapter 1 (Question 28; 5th edition)
8. Question 29 of Chapter 1 (Question 29; 5th edition)
   NOTE: If you are using the 5th edition, you also have to answer the following question. (c) Based on your observations in (b) of Question 29, explain why the plurality-with-elimination method satisfies the majority criterion.