Today’s main lesson:
• To learn two more fair-division methods.
  – The last-diminisher method
  – The method of markers

**The last-diminisher method**

Suppose that there are $N$ players who want to divide a booty $S$.

• **Preliminaries:** Before the players start dividing the booty $S$, they are randomly assigned an order of play. The order is as follows: player $P_1$ plays first, player $P_2$ plays second, and so on. The last player is $P_N$. The order is fixed until they finish dividing $S$.

○ **Round 1:**
  1. Player $P_1$ declares a share with $1/N$th worth of $S$ under $P_1$’s value system. Name this share $s_1$, and forward it to $P_2$.
  2. Player $P_2$ has two choices:
     • If $s_1$’s worth is **more** than $1/N$th worth of $S$ under $P_2$’s value system, then $P_2$ **diminishes** $s_1$ into a new $s_1$ so that the new $s_1$’s worth is $1/N$th worth of $S$ under $P_2$’s value system. Forward the (diminished) share $s_1$ to $P_3$. The “trimmed” part is returned to the remaining booty.
     • If $s_1$’s worth is **less than or equal to** $1/N$th worth of $S$ under $P_2$’s value system, then $P_2$ **passes** (does not diminish), and forwards the (undiminished) share $s_1$ to $P_3$.
  3. Players $P_3$ through $P_N$ follow exactly the same process as $P_2$ does ($P_N$ only needs to diminish $s_1$ or pass, but does not need to forward it.)
  4. Find **which player diminishes last** in round 1. This player, the **last diminisher** of round 1, gets the share $s_1$ and is eliminated. If there is no player who diminishes, the player who makes the first cut (in this case, player $P_1$) gets $s_1$, and is eliminated. If $P_1$ is eliminated, then the next remaining player in the order of play becomes the first player in the new order of play.

○ **Round 2:** The first player declares a share with $1/(N-1)$th worth of the remaining booty under that player’s value system (since there are now $N-1$ remaining players). Name this share $s_2$. The other remaining players diminish or pass as they do in round 1. That is, if a player thinks that $s_2$’s worth is more than $1/(N-1)$th worth of the remaining booty, that player diminishes $s_2$ so that the new $s_2$’s worth is $1/(N-1)$th worth of the remaining booty under that player’s value system. If not, then the player passes and forwards $s_2$ to the next player. The player who diminishes last in round 2 gets the share $s_2$ and is eliminated. If, on the other hand, all players pass, the player who makes the first cut gets $s_2$ and is eliminated.

○ **Round 3, 4, etc:** Repeat the process, each round having one less player and a smaller remaining booty, until there are only two players left. At this point, use the divider-chooser method among those two players to divide the remaining booty.
**The method of markers**

At times, a group of people may wish to divide *many* items rather than a *single* item. In this case the Method of Markers can be applied. Suppose that there are *N* players who want to divide *M* items (In this case, the booty *S* is *M* items).

- **Preliminaries:** The items are randomly arranged into an array.

  - **Step 1:** Player *P*₁ divides the array into *N* segments (by putting *N* − 1 markers from left to right along the array) so that each segment is a fair share according to *P*₁. Call the items between the left end of the array and *P*₁’s first marker “*P*₁’s first segment”, the items between the first marker and the second marker “*P*₁’s second segment” and so on. Independently, players *P*₂ through *P*ₙ do the same thing.

  - **Step 2:** Scan the array from left to right until the leftmost first marker is found. The player who placed this first marker will receive the items between the left end and that player’s first marker (that player’s first segment). That player’s markers are then removed, and the array is now missing these leftmost items. Scan again from left to right until the leftmost second marker is found. The player who placed that second marker will receive the items between his or her first marker and second marker (that player’s second segment). That player’s markers are then removed, and the array is now missing that player’s items. Repeat this process until each player has received a segment, with the final remaining player receiving the items from that player’s last marker to the right end of the array.

  - **Step 3:** The leftover items, if any, are divided among players by using any random draw method (thus, at times, the method of markers will result in players getting more than what they believe to be a fair share).

**Suggested problems for review**

You have no assignment this week. However, you are strongly recommended to solve the following questions to help review today’s lecture. Unfortunately, there are some questions written only in the 6th edition. If you have the 5th edition, please ask a friend who has the 6th edition, or borrow the 6th edition from the Douglas library so that you can photocopy the questions.

1. Question 41 of Chapter 3; 6th edition
2. Question 43 of Chapter 3; 6th edition
   (the *current C-piece* in Question 41 and 43 means “*s₁*” in the last-diminisher method.)
3. Question 45 of Chapter 3; 6th edition (Question 35 (a),(b),(d),(e) and (f) of Chapter 3; 5th edition)
4. Question 47 of Chapter 3; 6th edition
5. Question 59 of Chapter 3; 6th edition (Question 47 of Chapter 3; 5th edition)
6. Question 63 of Chapter 3; 6th edition (Question 51 of Chapter 3; 5th edition)
7. Question 65 of Chapter 3; 6th edition
8. Question 67 of Chapter 3; 6th edition