Today’s main lesson:

- To learn basic notions of graph theory.
  - some terminology
  - Euler’s theorems

**Some terminology**

**Graphs** are pictures consisting of dots called **vertices** and lines between vertices called **edges** (A single dot is called a **vertex**). For an edge, (one or two) vertices attached to the edge are called the **ends** of the edge. A **self-loop** or simply a **loop** is an edge to which only one vertex is attached (In other words, a loop is an edge which has only one end).

- Two **vertices** are said to be **adjacent** if there is an edge between those vertices.
- Two **edges** are said to be **adjacent** if they share the same vertex as an end.
- The **degree** of the vertex is the number of edges attached to it. A loop is counted as 2 edges. An **even vertex** is a vertex of even degree and an **odd vertex** is a vertex of odd degree.

- **Paths** are sequences of vertices satisfying the following:
  - each vertex in a path is adjacent to the next vertex
  - an edge can be used in a path **only once**
  
  The number of edges in a path is called the **length** of the path.

- **Circuits** are paths starting and ending at the same vertex.

- A graph is said to be **connected** if for any two vertices in the graph, there exists a path starting at one of them and ending at the other. If a graph is not connected (that is, if there are two vertices such that there is no path starting at one of them and ending at the other), the graph is said to be **disconnected**.

- For a “connected graph”, a **bridge** is an edge such that if the edge is removed from the graph, the resulting graph turns out to be a disconnected graph. Note that there are connected graphs which have no bridges.
Euler’s theorems

An Euler path is a path that passes through all edges of a graph. Like any other path, every edge can be used only once in an Euler path. An Euler circuit is a circuit that passes through all edges of a graph. As we recall, circuits are paths which start and end at the same vertex. Every edge can be used only once in an Euler circuit as well.

○ Euler’s theorem 1 (Euler’s circuit theorem):
(a) If a graph has an odd vertex, then the graph has no Euler circuits.
(b) If a graph is connected and every vertex is an even vertex, then the graph has an Euler circuit.

○ Euler’s theorem 2 (Euler’s path theorem):
(a) If a graph has 3 or more odd vertices, then the graph has no Euler paths.
(b) If a graph is connected and there are exactly two odd vertices, then the graph has an Euler path. In that case, any Euler path must start at one of those odd vertices and end at the other one.

○ Euler’s theorem 3 (Euler’s sum of degrees theorem):
(a) The sum of the degrees of all the vertices in a graph is equal to twice of the number of edges in the graph.
(b) A graph always has an even number of odd vertices.

Weekly Assignment 5 (Due: March 5th, 2008)

You may hand in your complete assignment at the next class (March 4th), or the following day at the math department office (Jeffery 310). Assignments should be stapled and clearly labeled with your full name, student number and the class number. There are some questions written only in the 6th edition. If you have the 5th edition, please ask a friend who has the 6th edition, or borrow the 6th edition from the Douglas library so that you can photocopy the questions.

1. Question 2 of Chapter 5; 6th edition
2. Question 3 of Chapter 5; 6th edition
4. Question 12 of Chapter 5; 6th edition (Question 12 of Chapter 5; 5th edition)
5. Question 19 of Chapter 5; 6th edition (Question 19 of Chapter 5; 5th edition)
7. Question 27 of Chapter 5; 6th edition (Question 29 of Chapter 5; 5th edition)
8. Question 34 of Chapter 5; 6th edition