Today’s main lesson:
- Trees and their properties
- Kruskal’s algorithm

**Trees and their properties**

A tree is a graph satisfying the following:
- A tree is a connected graph.
- A tree has no circuits.

**Property 1**
(a) If a graph is a tree, then for any pair of vertices in the graph, there is only one path starting with one vertex and ending with the other.
(b) If, for any pair of vertices in a graph, there is only one path starting with one vertex and ending with the other, then the graph is a tree.

**Property 2**
(a) If a graph is a tree, then every edge in the graph is a bridge. More precisely, if we delete an edge from a tree, then the resulting graph turns out to be a disconnected graph.
(b) If a graph is connected and every edge in the graph is a bridge, then the graph is a tree.

**Property 3**
(a) If a graph is a tree with $N$ vertices, then the graph has $N - 1$ edges.
(b) If a graph with $N$ vertices is connected and has $N - 1$ edges, then the graph is a tree.
(c) If a graph with $N$ vertices has no circuits and has $N - 1$ edges, then the graph is a tree.

**Kruskal’s algorithm**

Given a connected graph $G$ with $N$ vertices, we can find a tree which uses $N - 1$ edges and all $N$ vertices in $G$. Such a tree is called a spanning tree of $G$. If $G$ has weights on each edge (if $G$ is a connected weighted graph), then we can find a spanning tree $T$ with the smallest total weight among all spanning trees of $G$. Such a spanning tree $T$ is called a minimum spanning tree of $G$.

**Kruskal’s algorithm—an algorithm to find a minimum spanning tree:**
Given a connected weighted graph $G$ with $N$ vertices,

- **Step 1:** Among all edges in the connected weighted graph $G$, find and mark the edge with the smallest weight (if there are 2 or more edges with the smallest weight, pick one randomly and mark it).
- **Step 2, 3,..., $N - 1$:** Keep finding the edge with the smallest weight among unmarked edges and mark it, such that marked edges do not create a circuit. Stop this process when the marked edges and vertices attached to them have generated a spanning tree of $G$ (Step $N - 1$). That spanning tree is a minimum spanning tree of $G$. 


NOTE: GOOD NEWS: D Kruskal’s algorithm always gives us a minimum spanning tree of a connected weighted graph. Recall that the cheapest-link algorithm does not always give us an optimal Hamilton circuit of a complete weighted graph.

Weekly Assignment 7 (Due: March 19th, 2008)

   You may hand in your complete assignment at the next class (March 18th), or the following day at the math department office (Jeffery 310). Assignments should be stapled and clearly labeled with your full name, student number and the class number. There are some questions written only in the 6th edition. If you have the 5th edition, please ask a friend who has the 6th edition, or borrow the 6th edition from the Douglas Library so that you can photocopy the questions.

1. Question 1 of Chapter 7; 6th edition
2. Question 7 of Chapter 7; 6th edition (Question 7 of Chapter 7; 5th edition)
3. Question 10 of Chapter 7; 6th edition (Question 10 of Chapter 7; 5th edition)
4. Question 12 of Chapter 7; 6th edition
5. Question 18 of Chapter 7; 6th edition
6. Question 22 of Chapter 7; 6th edition (Question 22 of Chapter 7; 5th edition)
7. Question 23 of Chapter 7; 6th edition (Question 23 of Chapter 7; 5th edition)
8. Question 52 of Chapter 7; 6th edition