1) Suppose $f$ is a differentiable function on $\mathbb{R}$ with $f(0) = 1$ and that $f$ satisfies the equation $xf''(x) + f'(x) + xf(x) = 0$ (this is called a differential equation)

   i) find $f'(0)$;

   ii) find $f''(0)$.

2) Find the derivative of each the functions below.

   i) $f(x) = \sin^{-1}(\sqrt{\sin x})$, for $0 < x < \pi$.

   ii) $f(x) = \tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$.

3) Suppose that $f$ is differentiable and one-to-one with inverse $f^{-1}$. Moreover suppose that $f'(x) \neq 0$ for all $x$ and that $F$ is a differentiable function with $F' = f$. Let $G(x) = xf^{-1}(x) - F(f^{-1}(x))$. Show that $G'(x) = f^{-1}(x)$.

4) Consider the curve in the plane $y^3 - 3y = x$.

   i) find the points where the curve crosses the $y$-axis;

   ii) find the slope of the tangent line at each of these points.