MATH 120
Differential and Integral Calculus
Autumn 2013

Assignment 7, Due November 1

1) Find an anti-derivative for each of the given functions:
   \( i) \ f(x) = x^2 \sqrt{3 + x^3}, \)
   \( ii) \ f(x) = x^5 \sqrt{3 + x^3}, \)
   \( iii) \ f(x) = \sin^3(x). \)

Check that your answer is correct by differentiating and briefly explain how you got your anti-derivative.

2) Using the properties for definite integrals, but without evaluating the integrals, show that:
   \( i) \ \sqrt{\frac{2\pi}{24}} \leq \int_{\pi/6}^{\pi/4} \cos(x) \, dx \leq \sqrt{\frac{3\pi}{24}}; \)
   \( ii) \ \int_0^1 \sqrt{1 + x^2} \, dx \leq \int_0^1 \sqrt{1 + x} \, dx. \)

3) By interpreting \( \frac{1}{n} \sum_{i=1}^{n} \frac{1}{1 + (i/n)^2} \) as a Riemann sum, write \( \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{1 + (i/n)^2} \) as a definite integral.