Instructions: This is a 50-minute quiz. There are 4 questions worth a total of 20 marks as indicated in the box below. Answer all questions in the space provided. If you need more room, answer on the back of the previous page. Show all your work and explain how you arrived at your answers, unless explicitly told to do otherwise. Except where a decimal answer is asked for, it is preferable to leave answers in the form $\sqrt{\pi}$, $e^2$, etc. However, do any obvious simplification. For example, $2 + \frac{1}{2} + \frac{1}{3} = 2\frac{5}{6}$ or $\frac{17}{6}$, and $\frac{(x + 1)^2}{x + 1} = x + 1$. Only CASIO FX-991 or Gold/Bule Sticker calculators are permitted.

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Problem 1: Solve the following equation for $x$.

$$\ln(x + 4) = 3 + \ln(x - 2)$$

Solution: We have

$$\ln(x + 4) - \ln(x - 2) = 3$$

Applying laws of logarithms, we obtain

$$\ln\left(\frac{x + 4}{x - 2}\right) = 3$$

Then

$$\frac{x + 4}{x - 2} = e^3$$

which is equivalent to

$$x + 4 = e^3(x - 2)$$

So

$$x = \frac{4 + 2e^3}{e^3 - 1}$$
Problem 2: Let \( p = \frac{3}{100}q + 3 \) be the supply equation for a manufacturer’s product, and suppose the demand equation is \( p = -\frac{1}{150}q + 14 \), where \( p \) represents price per unit in dollars and \( q \) represents the number of units sold per time period.

(a) Draw the graphs of the supply and demand equations on the coordinate plane below.

(b) Find the equilibrium price and mark the equilibrium point on your graph above.

Solution: To find the equilibrium point, we solve the following system of equations.

\[
\begin{align*}
\begin{cases}
 p &= \frac{3}{100}q + 3 \\
 p &= -\frac{1}{150}q + 14
\end{cases}
\end{align*}
\]

We have

\[
\frac{3}{100}q + 3 = -\frac{1}{150}q + 14
\]

It follows that

\[
\frac{3}{100}q + \frac{1}{150}q = 14 - 3
\]

that is,

\[
\frac{9q + 2q}{300} = 11
\]

So \( q = 300 \). Then the equilibrium price is \( p = 12 \).
**Problem 3:** The demand function of a certain product is

\[ p = 140 - q \]

where \( p \) is the price per unit and \( q \) is the number of units sold in a month. Suppose there is a fixed cost of $3750 to set up the product and each unit costs $15 to produce.

(a) What is the total revenue \( R(q) \)?

**Solution:** The total revenue is

\[ R(q) = pq = (140 - q)q = 140q - q^2. \]

(b) What is the total cost \( C(q) \)?

**Solution:** The total cost is \( C(q) = 3750 + 15q \).

(c) What is the profit \( P(q) \)?

**Solution:** The profit is

\[ P(q) = R(q) - C(q) = 140q - q^2 - 3750 - 15q = -q^2 + 125q - 3750 \]

(d) Find the break-even point(s).

**Solution:** The break-even occurs when \( P(q) = 0 \), i.e., \( -q^2 + 125q - 3750 = 0 \). Solving this quadratic equation, we obtain \( q_1 = 50 \) and \( q_2 = 75 \). Then \( p_1 = 90 \) and \( p_2 = 65 \). So break even points are \((50, 90)\) and \((75, 65)\).
Problem 4: Suppose that $500 amounted to $588.38 in a saving account after three years. If the interest was compounded semiannually,

(a) Find the nominal rate of interest (A.P.R.).

Solution: Let \( r \) be the nominal rate, \( S \) the compounded amount and \( P \) the principal. In there years, there are 6 interest period. We have

\[
S = P \left( 1 + \frac{r}{2} \right)^6
\]

where \( S = 588.38 \) and \( P = 500 \). To find the nominal rate, we solve the following equation for \( r \),

\[
588.38 = 500 \left( 1 + \frac{r}{2} \right)^6
\]

\[
1 + \frac{r}{2} = \left( \frac{588.38}{500} \right)^{1/6}
\]

So the nominal rate is

\[
r = 2 \left[ \left( \frac{588.38}{500} \right)^{1/6} - 1 \right] \approx 0.055
\]

(b) How many months does it take for the money to double? (please round your answer to one decimal place).

Solution: Let \( t \) be the number of interest periods for the money to double. Since A.P.R is compounded semiannually, the periodic rate is \( 0.055/2 = 0.0275 \). Then after \( t \) interest periods,

\[
500(1 + 0.0275)^t = 1000
\]

It implies

\[
(1 + 0.0275)^t = 2
\]

Solving it for \( t \), we obtain

\[
t = \frac{\ln 2}{\ln 1.0275} = 25.55 \text{ interest periods}
\]

Note that there are 6 months in an interest period. The number of months it takes for the money to double is

\[
25.55 \times 6 \approx 153.3 \text{ months}
\]