

The Euclidean Algorithm

First and Second Version

First Version: By using alternate subtraction:

$$\begin{array}{r|l|l|l|l|l|l}
 143 & 143 & 91 & 39 & 39 & 26 & 13 \\
 \hline
 195 & 52 & 52 & 52 & 13 & 13 & 13
 \end{array}$$

Second Version: By using the division algorithm:

$$\begin{aligned}
 195 &= 1 \cdot 143 + 52 \\
 143 &= 2 \cdot 52 + 39 \\
 52 &= 1 \cdot 39 + 13 \\
 39 &= 3 \cdot 13 + 0
 \end{aligned}$$

Procedure: Given: integers $m, n \neq 0$.

Step 1: Put $r_{-1} = m, r_0 = n$.

Step 2: Define successively, for $i = 1, 2, \dots, k$:

$$r_i = \text{rem}(r_{i-2}, r_{i-1}).$$

Stop when $r_i = 0$. (Thus: $\text{rem}(r_{k-1}, r_k) = 0$.)

Result: $r_k = \text{gcd}(m, n)$.