The General Solution of the Diophantine Equation

$$mx + ny = c$$

Theorem 5: Let $m, n, c \in \mathbb{Z}$ be non-zero integers and suppose that

$$g := \gcd(m, n) \mid c.$$

Then the general integer solution (x, y) of the equation

(1) $$mx + ny = c$$

is given by the formula

(2) $$\left.\begin{aligned} x &= \tfrac{c}{g}x_0 + \tfrac{n}{g}t \\ y &= \tfrac{c}{g}y_0 - \tfrac{m}{g}t \end{aligned}\right\} \text{ where } t \in \mathbb{Z},$$

and $x_0, y_0 \in \mathbb{Z}$ are (any) integers satisfying the equation

(3) $$mx_0 + ny_0 = g.$$