How to Solve $mx + ny = c$

Step 0: Use the Euclidean algorithm to find
\[ g := \gcd(m, n). \]

Step 1: Use the GCD-criterion to check whether the equation

(1) \[ mx + ny = c \]

has any solutions; i.e. test whether

(2) \[ g := \gcd(m, n) \mid c. \]

If false, then equation (1) has no integer solutions.

Step 2: Use the method of back-substitution in the Euclidean algorithm to find integers \((x_0, y_0)\) such that

(3) \[ mx_0 + ny_0 = g. \]

Step 3: The general solution \((x, y)\) of (1) is given by

(4) \[
\begin{align*}
x &= \frac{c}{g}x_0 + \frac{n}{g}t \\
y &= \frac{c}{g}y_0 - \frac{m}{g}t
\end{align*}
\]
where \(t \in \mathbb{Z}\).

Step 4: If applicable, analyze the constraints.
(E.g. \(x \geq 0, y \geq 0 \leadsto \) inequalities for \(t\).)