The Wheel Problem

Problem:

Given: 3 wheels:
A : radius 35cm
B : radius 25cm
C : radius 5cm

What is the minimal number of rotations of C that are necessary such that A points to 2 and B points to 3?

Analysis: If C rotates once, a pt. on C moves 5(2π)cm
⇒ a point on A moves 5 · 2π = \frac{1}{7}35(2\pi)cm
⇒ A makes \frac{1}{7} rot’n ⇒ A’s pointer advances by 1.
Similarly: If C rotates once, a point on B moves
5 · 2π = \frac{1}{5}25(2π)cm ⇒ B makes \frac{1}{5} rotation
⇒ B’s pointer advances by 1.

Thus: if x denotes a fixed number of rotations of C, then for x = 0, 7, 14, . . . , A’s pointer is at 0, and for x = 2, 9, 16, . . . , A’s pointer is at 2.
Therefore: x ≡ 2 (mod 7) ⇔ A’s pointer is at 2.
Similarly: x ≡ 3 (mod 5) ⇔ B’s pointer is at 3.

Result: \begin{align*}
\{ A’s \text{ pointer is at } 2 \} &\iff \{ x \equiv 2 \pmod{7} \} \\
\{ B’s \text{ pointer is at } 3 \} &\iff \{ x \equiv 3 \pmod{5} \}
\end{align*}