Irreducible Polynomials

**Definition:** A polynomial $f(x) \in F[x]$ is called irreducible over $F$ if $\deg(f) > 0$ and if its only factors are $c$ and $cf(x)$, where $c \in F, c \neq 0$, is any non-zero constant. If $\deg(f) > 0$ and $f$ is not irreducible, then $f$ is called reducible over $F$.

**Note:** If $\deg(f) > 0$, then $f$ is reducible over $F \iff$ there exists $g \in F[x]$ with $0 < \deg(g) < \deg(f)$ and $g | f$.

**Theorem 7** a) Every linear polynomial $f(x) = x - a \in F[x]$ is irreducible in $F[x]$.  
   
b) If $f$ is irreducible in $F[x]$ and $\deg(f) \geq 2$, then $f(a) \neq 0$, for all $a \in F$.  
c) If $\deg(f) = 2$ or $3$, then $f$ is irreducible in $F[x]$ if and only if $f(a) \neq 0$, for all $a \in F$.

**Remark.** The theorem shows that there is a partial relationship between the following two concepts:  
(i) $f(x)$ is reducible in $F[x]$;  
(ii) $f(x)$ has a root in $F[x]$.  
Indeed, we have $(ii) \iff (i)$ if $\deg(f) = 2$ or $3$, but in general we only have $(ii) \Rightarrow (i)$.