Problem 6(a) : Using Maple's power mod command:

\[
> \text{Power}(1234, 123456789) \mod 5555; \\
> 1799
\]

Thus, MAPLE found that \( \text{rem}(1234^{123456789}, 5555) = 1799 \).

If we try to compute this in the naive way, then we get an overflow error:

\[
> \text{modp}(1234^{123456789}, 5555); \text{irem}(1234^{123456789}, 5555);
\]

Error, cannot reallocate memory (old size=8 new size=162037064)

The reason for this is that MAPLE first tries to compute \( 1234^{123456789} \), which is a gigantic number. (It is greater than \( 10^{27} \).)

Problem 6(b) : Using Maple's chrem(..) command to solve simultaneous congruences:

First system:

\[
> a := [2, 3, 5, 6]; m := [4, 5, 7, 9];
\]

Thus, the solution to the simultaneous congruences is \( x = 978 \mod(4\cdot5\cdot7\cdot9) \). This is correct because

\[
> [\text{seq}(\text{modp}(x, m[i]), i = 1..4)];
\]

\[
[2, 3, 5, 6]
\]

which is the of values of \( a \). A better way to check this is by using the evalb command:

\[
> \text{evalb}([\text{seq}(\text{modp}(x, m[i]), i = 1..4)] = a);
\]

\[
\text{true}
\]

Second system:

Since this system is not in the form \( x = a_1 \mod(m_1), ..., x = a_r \mod(m_r) \), we first have convert the system to this form. There are two ways to do this:

First solution: Manually convert each congruence to the desired form:

For example, since \( 3 \cdot 3 = 1 \mod(4) \), the equation \( 3x = 2 \mod(4) \) is equivalent to \( x = 6 \mod(4) \). Similarly, \( 2x = 3 \mod(5) \) is the same as \( x = 9 \mod(5) \), and \( 4x = 5 \mod(7) \) becomes \( x = 10 \mod(7) \) and \( 5x = 6 \mod(9) \) means \( x = 12 \mod(9) \).

Thus, we apply chrem with

\[
> a := [6, 9, 10, 12];
\]

\[
a := [6, 9, 10, 12]
\]

\[
> y := \text{chrem}(a, m);
\]

\[
y := 1074
\]

This is the correct solution because

\[
> [\text{modp}(3\cdot y, 4), \text{modp}(2\cdot y, 5), \text{modp}(4\cdot y, 7), \text{modp}(5\cdot y, 9)];
\]

\[
[2, 3, 5, 6]
\]

which is the coefficient vector on the right hand side of the original second system.

Second solution: use chrem with fractions

For example, dividing both sides of \( 3x = 2 \mod(4) \) by \( 3 \) yields \( x = 2/3 \mod(4) \), and similarly \( 2x = 3 \mod(5) \) is the same as \( x = 3/2 \mod(5) \), etc. Thus:
\texttt{> chrem}\left(\left[\frac{2}{3}, \frac{3}{2}, \frac{5}{4}, \frac{6}{5}\right], m\right);

\texttt{>}

\texttt{1074}

\texttt{>}

\texttt{gives the same answer as before.}