The Lagrange Interpolation Polynomial

**Theorem 3 (“Matrix Method”)**

The coefficients $a_0, a_1, \ldots, a_{n-1}$ of the unique interpolation polynomial

$$f(x) = \sum_{k=0}^{n-1} a_k x^k$$

of degree $\leq n - 1$ which passes through the data points $(x_1, y_1), \ldots, (x_n, y_n)$ is given by the following system of linear equations

$$
\begin{pmatrix}
1 & x_1 & x_1^2 & \ldots & x_1^{n-1} \\
1 & x_2 & x_2^2 & \ldots & x_2^{n-1} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & x_n & x_n^2 & \ldots & x_n^{n-1}
\end{pmatrix}
\begin{pmatrix}
a_0 \\
a_1 \\
\vdots \\
a_{n-1}
\end{pmatrix}
=
\begin{pmatrix}
y_1 \\
y_2 \\
\vdots \\
y_n
\end{pmatrix}.
$$

**Note:** The above matrix $A$ is called the **Vandermonde matrix** defined by $x_1, x_2, \ldots, x_n$. Its determinant is called the **Vandermonde determinant**:

$$V(x_1, x_2, \ldots, x_n) = \prod_{1 \leq i < j \leq n} (x_j - x_i).$$