Solving Discrete Linear Systems

**Theorem 15:** The discrete linear system with constant coefficients

\[ \vec{u}_{n+1} = A\vec{u}_n + \vec{b}_n, \quad n = 0, 1, \ldots \]

has the (unique) solution

\[ \vec{u}_n = A^n\vec{u}_0 + A^{n-1}\vec{b}_0 + A^{n-2}\vec{b}_1 + \ldots + A\vec{b}_{n-2} + \vec{b}_{n-1}. \]

In particular, if the discrete linear system (1) is homogeneous (i.e. \( \vec{b}_n = \vec{0} \) for all \( n \)), then

\[ \vec{u}_n = A^n\vec{u}_0. \]

**Method for solving discrete linear systems:**

**Step 1:** Use Theorem 12 to write down the general formula for \( \vec{u}_n \) in terms of powers of \( A \).

**Step 2:** Derive an explicit formula for \( A^n \). (Use the theory of matrix polynomials.)

**Note:** To solve a difference equation, first transform it into a discrete linear system and then solve this system by using the above method.