The Invariance Property

Theorem 3: (Invariance Property) If $B = P^{-1}AP$, then

$$\begin{aligned} \mathrm{ch}_A(t) &= \mathrm{ch}_B(t), \\ E_A(\lambda) &= PE_B(\lambda) \;:=\; \{P\vec{v} : \vec{v} \in E_B(\lambda)\}. \end{aligned}$$

In particular, we have

$$\begin{aligned} m_A(\lambda) &= m_B(\lambda), \\ \nu_A(\lambda) &= \nu_B(\lambda). \end{aligned}$$

Example. Let $B = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$, $P = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{pmatrix}$,

and $A = PBP^{-1} = \begin{pmatrix} 4 & 0 & -2 \\ 1 & 2 & -1 \\ 2 & 0 & 0 \end{pmatrix}$.

Then:
$$\begin{aligned} E_B(2) &\overset{Th.2}{=} \{c_1(1,0)^t\} \oplus \{c_2(1)\} \\ &= \{c_1(1,0,0)^t + c_2(0,0,1)^t\} \\ \Rightarrow E_A(2) &\overset{Th.3}{=} \{c_1 \underbrace{P(1,0,0)^t}_{\substack{1^{st}\text{ column} \\ \text{of } P}} + c_2 \underbrace{P(0,0,1)^t}_{\substack{3^{rd}\text{ column} \\ \text{of } P}}\} \\ &= \{c_1(2,1,2)^t + c_2(1,1,1)^t\}. \end{aligned}$$