Shipping of Commodities

**Situation:** Four regions (or cities) $R_1, R_2, R_3, R_4$ ship (non-renewable) commodities (such as antique paintings, rental cars) among themselves according to the following diagram:

The figures represent the percentage of goods each region ships per week. (Percentage: in terms of goods present)

**Today’s distribution:**
- $R_1$ has $300,000$ worth of goods
- $R_2$ has $200,000$ worth of goods
- $R_3$ has $100,000$ worth of goods
- $R_4$ has $100,000$ worth of goods

**Question:** What happens in the long run?

**Analysis:** This is a discrete linear system $\vec{v}_{n+1} = A\vec{v}_n$ with
\[ A = \frac{1}{10} \begin{pmatrix} 1 & 3 & 0 & 0 \\ 7 & 6 & 0 & 0 \\ 1 & 0 & 10 & 0 \\ 1 & 1 & 0 & 10 \end{pmatrix} = \begin{pmatrix} T & 0 \\ B & I \end{pmatrix}, \quad \vec{v}_0 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}. \]