Math 211
Assignment 1

Due 29 September 2017

Do the following problems by hand (and show your work). You may check your results (of the computational problems) by MAPLE, if you wish. For the questions requiring explanations or proofs, remember to write clear sentences.

Hand in your MAPLE output for problems 6 and 7. Remember to use the Worksheet option. In addition, remember to use MAPLE’s “text feature” to put your name on the output and to insert your comments before and after your computations. See the handout “MAPLE Homework Instructions” for further instructions.

1. Use the Euclidean algorithm to reduce the following fractions to their lowest terms.
   (a) \( \frac{144}{804} \); (b) \( \frac{1245}{2235} \).
   In part (a), use both methods and comment on which method you prefer. In part (b), use the method of your choice.

2. Use the Euclidean algorithm to find the greatest common divisor of:
   (a) 39617 and 41449; (b) 12553 and 22307.

3. Convert the improper fraction \( \frac{1000}{13} \) to a mixed number. (Explain your method.)

4. How many integers between 600 and 2000 are divisible by 17? Justify your answer.

5. Use induction to prove that \( n^3 - n \) is divisible by 6 for every positive integer \( n \).

6. Use the MAPLE command “igcd(\( a, b \))” to find the greatest common divisor of
   (i) 12345 and 54321; (ii) 213141516171 and 262524232221.

7. (a) Let \( L \) be the list whose \( k \)-th entry is \( \text{gcd}(k^2, 24) \) for \( 1 \leq k \leq 12 \). Use the MAPLE command \([\text{seq}(\ldots)]\) to construct this list, and use a suitable MAPLE command to determine the 10\(^{th} \) element of this list.

   (b) Let \( LL \) be the list whose \( k \)-th entry is the ordered pair \((k, \text{gcd}(k^2, 12))\) for \( 1 \leq k \leq 12 \). Use the MAPLE command \([\text{seq}(\ldots)]\) to construct this list of lists. Find \( LL[9] \) and \( LL[9,2] \).

   (c) Write a one-line function \( f(x) \) which calculates \( x^2 - x + 1 \). Use your function to determine \( f(20) \).