1. Use the GCD-criterion to decide which of the following equations has an integral solution \( x, y \in \mathbb{Z} \). Furthermore, find such a solution for each equation which is solvable.

   (a) \( 294x + 816y = -9 \);
   (b) \( 777x - 1628y = 37 \);
   (c) \( 1233x + 2115y = 18 \).

2. Let \( s, t \in \mathbb{Z} \) and put 
\[
x = t^2 - s^2, \quad y = 2st \quad \text{and} \quad z = t^2 + s^2.
\]
   (a) Verify that \((x, y, z)\) is a Pythagorean triple, i.e. that \(x^2 + y^2 = z^2\).
   (b) Verify that for \( t = s + 1 \) we get the triples \((2s + 1, 2s(s + 1), 2s(s + 1) + 1)\). What are the triples corresponding to the values of \( s = 1, 2, 3, 4 \)?

3. Is each of the following linear Diophantine equations solvable in integers \( x, y \in \mathbb{Z} \)? If so, write down the general solution. If not, explain why not.

   (a) \( 8023x - 8249y = 1243 \);
   (b) \( 1079x + 1411y = 243 \);
   (c) \( 123456x - 654321y = 7 \).

4. MAPLE problem (refer to the MAPLE instruction sheet):

   (a) Use the MAPLE command \( \text{igcdex}() \) to find \( \gcd(m, n) \) and integers \( x \) and \( y \) such that \( mx + ny = \gcd(m, n) \) when \( m = 1234567 \) and \( n = 5474970 \). Explain your output and verify that your \( x \) and \( y \) satisfy the given equation.

   (b) Consider the following MAPLE commands:
\[
euclid:= \text{proc}(m,n) \text{ local } q,r1,r2,r3; \
r1:=m; r2:=n; \
\text{while } r2 <> 0 \text{ do; } \
\quad q:=\text{iquo}(r1,r2); r3:= \text{irem}(r1,r2); \
\quad \text{lprint}(r1, '\text{='}, q, '\text{*}', r2, '\text{+}', r3); \
\quad r1:=r2; r2:=r3; \text{od; } \
\text{return}(r1); \text{ end;}
\]
Implement this program on the computer and explain what the program does. (Use the “text insertion” feature of MAPLE.) Also, test the program on your favourite pair of 4-digit numbers (your choice). In addition, test it for \( m = 1122344556677889977 \) and \( n = 9753124680123456789 \).