Math 211

Assignment 9

Due 1 December 2017

1. Find the remainder when $x^{1999} + x$ is divided by:
   (a) $2x + 1$;  
   (b) $x^2 - 3x + 2$;  
   (c) $x + \frac{-1+i\sqrt{3}}{2}$;  
   (d) $x^2 + x + 1$.

2. (a) Use the Euclidean algorithm to find the gcd $g(x)$ of $f_1(x) = 4x^4 + x^2 - x - 1$ and $f_2(x) = 4x^3 + 4x^2 + x$.
   (b) Use the method of back-substitution to determine polynomials $h_1(x)$ and $h_2(x) \in \mathbb{Q}[x]$ such that $f_1h_1 + f_2h_2 = g$.

3. Prove that the following polynomials are irreducible over the given field:
   (i) $x^2 - x + 2 \in \mathbb{Q}[x]$;  
   (ii) $x^2 - 2 \in \mathbb{F}_5[x]$;  
   (iii) $x^3 - 2 \in \mathbb{F}_7[x]$.

4. Let $f(x) = (x-1)^3(x^2+x-2)^2(x^2+x+1)^3$ and $g(x) = (x+2)^2(x^2-3x+2)^3(x^2+x+1)^4$.
   (a) What are the roots of $f$ in $\mathbb{C}$, and the multiplicities of each?
   (b) Find the gcd of $f$ and $g$ without using the Euclidean algorithm.

5. (a) Show that $f(x) = x^2 + 1$ is reducible over $\mathbb{F}_5$ but is irreducible over $\mathbb{F}_7$.
   (b) More generally, show that $f(x)$ is irreducible over $\mathbb{F}_p$ if $p \equiv 3 \pmod{4}$.
   [Hint: show that the existence of a root would contradict Fermat’s Theorem.]

6. Express $f(x) = x^5 - 1$ as a product of irreducible factors in $\mathbb{C}[x]$. (You can leave the coefficients of your answer in terms of sines and cosines, if you want.)