Math 211

Assignment 11

Due 26 January 2018

[4] 1. For each of the following two sets of data points, find the parabola \( y = a_0 + a_1 x + a_2 x^2 \) which best fits these points in the vertical least square sense. Also, find the minimal mean square deviation in each case.

(a) \((0, 2), (1, 3), (-1, 2), (2, 2)\);

(b) \((0, 1), (1, 2), (-1, 4), (2, 7)\).

[3] 2. Consider the set \( L \) of points \((x, y, z)\) in \( \mathbb{R}^3 \) defined by the equation \( 2y + z = 2 \). Show that \( L \) is a plane by finding a vector equation for \( L \). Also, find a normal vector to \( L \).

[3] 3. Let \( S \subset \mathbb{R}^5 \) be the solution set of the system of equations

\[
\begin{align*}
x_2 + 2x_3 + x_5 & = 1 \\
x_3 + x_5 & = 1 \\
x_3 + x_4 & = 1
\end{align*}
\]

Verify that \( S \) is a linear set in \( \mathbb{R}^5 \) by finding a vector equation for it. What is the dimension of \( S \)?

[4] 4. (a) Let \( a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n \in \mathbb{R} \) be real numbers. By using a suitable theorem discussed in class, prove that

\[
(a_1 b_1 + a_2 b_2 + \ldots + a_n b_n)^2 \leq (a_1^2 + \ldots + a_n^2)(b_1^2 + \ldots + b_n^2).
\]

When does equality hold?

(b) Use part (a) to show that for any positive real numbers \( c_1, c_2, \ldots, c_n > 0 \) we have

\[
\frac{1}{c_1} + \ldots + \frac{1}{c_n} \geq \frac{n^2}{c_1 + \ldots + c_n}.
\]

[6] 5. Let \( V = \langle \vec{v}_1, \ldots, \vec{v}_4 \rangle \) be the subspace of \( \mathbb{R}^5 \) spanned by the vectors \( \vec{v}_1 = (1, 0, 1, 2, 0), \vec{v}_2 = (-1, 1, -1, 0, 0), \vec{v}_3 = (1, 0, 1, 0, 1), \) and \( \vec{v}_4 = (0, 3, 0, 4, 1) \in \mathbb{R}^5 \).

(a) By row-reducing a suitable matrix, show that the vectors \( \{ \vec{v}_1, \vec{v}_2, \vec{v}_3 \} \) form a basis of \( V \).

(b) Calculate the matrix of the orthogonal projection operator \( P_V \).

(c) Find the unique vector \( \vec{v} \in V \) which is closest to the vector \( \vec{y} := (1, 1, 1, 1, 1) \), and express \( \vec{v} \) as a linear combination of the basis vectors \( \vec{v}_1, \vec{v}_2 \) and \( \vec{v}_3 \).

(d) Find the distance of the point \( \vec{y} \) to the subspace \( V \).