Do the following problems by hand (and show your work).

1. Let \( f, g \in F[x] \) (where \( F = \mathbb{Q}, \mathbb{R} \) or \( \mathbb{C} \)) be two polynomials with \( g \neq 0 \) and let \( c \in F, \ c \neq 0 \). Prove that
   \[
   \text{quot}(f, cg) = \frac{1}{c} \text{quot}(f, g) \quad \text{and} \quad \text{rem}(f, cg) = \text{rem}(f, g).
   \]

2. Let \( f, g \in F[x] \) (where \( F = \mathbb{Q}, \mathbb{R} \) or \( \mathbb{C} \)) be monic polynomials of the same degree. Prove that \( f | g \) if and only if \( f = g \).

3. Find the remainder when \( x^{1999} + x \) is divided by:
   (a) \( 2x + 1 \);
   (b) \( x^2 - 3x + 2 \);
   (c) \( x + \frac{-1+i\sqrt{3}}{2} \);
   (d) \( x^2 + x + 1 \).

4. (a) Use the Euclidean algorithm to find the gcd \( g(x) \) of \( f_1(x) = 4x^4 + x^2 - x - 1 \) and \( f_2(x) = 4x^3 + 4x^2 + x \).
   (b) Use the method of back-substitution to determine polynomials \( a(x) \) and \( b(x) \in \mathbb{Q}[x] \) such that \( af_1 + bf_2 = g \).

5. (a) Prove that \( x^2 - x + 2 \) is irreducible over \( \mathbb{Q} \).
   (b) Prove that \( x^2 - x + 2 \) is NOT irreducible over \( \mathbb{C} \).

6. Let \( f(x) = (x-1)^3(x^2+x-2)^2(x^2+x+1)^3 \) and \( g(x) = (x+2)^3(x^2-3x+2)^3(x^2+x+1)^4 \).
   (a) What are the roots of \( f \) in \( \mathbb{C} \), and the multiplicities of each?
   (b) Find the gcd of \( f \) and \( g \) without using the Euclidean algorithm.

7. Express \( f(x) = x^5 - 1 \) as a product of irreducible factors in \( \mathbb{C}[x] \). (You can leave the coefficients of your answer in terms of sines and cosines, if you want.)