1. (a) The first version (table method) of the Euclidean algorithm yields:

<table>
<thead>
<tr>
<th>144</th>
<th>144</th>
<th>144</th>
<th>144</th>
<th>144</th>
<th>60</th>
<th>60</th>
<th>36</th>
<th>12</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>804</td>
<td>660</td>
<td>516</td>
<td>372</td>
<td>228</td>
<td>84</td>
<td>84</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
</tbody>
</table>

The second version of the Euclidean algorithm (using the division algorithm) gives

\[
\begin{align*}
804 &= 5 \cdot 144 + 84 \\
144 &= 1 \cdot 84 + 60 \\
84 &= 1 \cdot 60 + 24 \\
60 &= 2 \cdot 24 + 12 \\
24 &= 2 \cdot 12 + 0
\end{align*}
\]

Thus, by both methods we see that \( \gcd(144, 804) = 12 \). Note that the first version required 10 steps (subtractions), whereas the second version needed only 5 steps (divisions). Thus, the second method is better since there are fewer steps.

Since \( 144 \div 12 = 12 \) and \( 804 \div 12 = 67 \), we obtain

\[
\frac{144}{804} = \frac{12}{67},
\]

which is in lowest terms.

(b) The second versions of the Euclidean algorithm yields

\[
\begin{align*}
2235 &= 1 \cdot 1245 + 990 \\
1245 &= 1 \cdot 990 + 255 \\
990 &= 3 \cdot 255 + 225 \\
255 &= 1 \cdot 225 + 30 \\
1225 &= 7 \cdot 30 + 15 \\
225 &= 2 \cdot 15 + 0
\end{align*}
\]

Thus, \( \gcd(1245, 2235) = 15 \). Since \( 1245 \div 15 = 83 \) and \( 2235 \div 15 = 149 \), we see that \( \frac{1245}{2235} = \frac{83}{149} \), which is in lowest terms.

2. (a) The Euclidean algorithm gives:

\[
\begin{align*}
41449 &= 1 \cdot 39617 + 1832 \\
39617 &= 21 \cdot 1832 + 1145 \\
1832 &= 1 \cdot 1145 + 687 \\
1145 &= 1 \cdot 687 + 458 \\
687 &= 1 \cdot 458 + 229 \\
458 &= 2 \cdot 229
\end{align*}
\]

Thus, \( \gcd(41449, 39617) = 229 \).

(b) Here the Euclidean algorithm yields:

\[
\begin{align*}
22307 &= 1 \cdot 12553 + 9754 \\
12553 &= 1 \cdot 9754 + 2799 \\
9754 &= 3 \cdot 2799 + 1357 \\
2799 &= 2 \cdot 1357 + 85 \\
1357 &= 15 \cdot 85 + 82 \\
85 &= 1 \cdot 82 + 3 \\
82 &= 27 \cdot 3 + 1 \\
82 &= 3 \cdot 1
\end{align*}
\]

Thus, \( \gcd(22307, 12553) = 1 \).

3. By the division algorithm we have \( 1000 = 76 \cdot 13 + 12 \). Thus, dividing both sides by 13 we obtain \( \frac{1000}{13} = 76 + \frac{12}{13} \), and so \( \frac{1000}{13} = 76 \frac{12}{13} \).
4. Since $2000 = 117 \cdot 17 + 11$, there are 117 numbers between 1 and 2000 which are divisible by 17. Of these, 35 are less than 600 (because $600 = 35 \cdot 17 + 5$), and so there are $82 = 117 - 35$ numbers between 2000 and 600 which are multiples of 17.

5. Let $P(n)$ be the statement: “$n^3 - n$ is divisible by 6.” Then $P(1)$ is true since $6|1^3 - 1$. Thus, assume $P(k)$ is true for some $k \geq 1$, i.e. that $6|(k^3 - k)$; we want to show that $P(k + 1)$ is also true. Now $(k + 1)^3 - (k + 1) = (k^3 + 3k^2 + 3k + 1) - (k + 1) = k^3 + 3k^2 + 2k + (k^3 - k) = (k^3 - k) + 3(k^2 + k)$. We observe that $k^2 + k = k(k + 1)$ is always even (because one of $k$ and $k + 1$ is always even), so we can write $k^2 + k = 2h$ with $h \in \mathbb{Z}$. (Alternately, we could use Example 1.7 of the text see that $k^2 + k = (k^2 - k) + 2k$ is always even.) Thus, we have $(k + 1)^3 - (k + 1) = k^3 - k + 6h$. By the induction hypothesis, $6|(k^3 - k)$, and hence $6|(k^3 - k + 6h) = ((k + 1)^3 - (k + 1))$, which means that $P(k + 1)$ is also true. Thus, $P(n)$ is true for every positive integer $n \geq 0$.

6. (See the MAPLE solution on the course web site for more details.)

The MAPLE command “\texttt{igcd(12345, 54321);}” returns the value 3, so gcd(12345, 54321) = 3. Similarly, gcd(213141516171, 262524232221) = 3.

7. (See course web site).

(a) The list L of elements gcd(k, 24) is defined by the command \texttt{L:= [seq(igcd(k^2,24), k=1..24)];} and Maple prints \texttt{L:= [1, 4, 3, 8, 1, 12, 1, 8, 3, 4, 1, 24].} The 10th element of this list is given by the command \texttt{L[10];} and Maple returns the value 4.

(b) The list LL of pairs (k, gcd(k,12)) is given by the command \texttt{LL:= [seq([k,igcd(k^2,12)], k=1..12)];} Maple computes \texttt{LL:= [[1,1], [2,4], [3,3], [4,4], [5,1], [6,12]], [7,1], [8,4], [9,3], [10,4], [11,1], [12,12]].} The commands \texttt{LL[9];} and \texttt{LL[9,2];} yield [9,3] and 3, respectively. Thus, the 9th element of LL is the list [9,3], and the 2nd element of this list is 3.

(c) The function f is defined by \texttt{f:= x -> x^2 - x + 1;} and the command \texttt{f(10);} computes the value of f at $x = 20$. The answer is $f(20) = 20^2 - 20 + 1 = 381$.

\textbf{Note:} Remember to put your name (in Maple text) at the beginning of your Maple homework and to add comments to explain your Maple output, as was done above.