Problem 1:

(a) False.

(b) False (since $q \rightarrow r$ is false).

(c) False.

(d) False (since $s \rightarrow (p \land \neg r)$ is false).

Problem 2:

(a) $p \rightarrow (q \lor r)$, where $p$ is “$x$ is prime”, $q$ is “$x$ is odd”, and $r$ is “$x = 2$”.

(b) $p \rightarrow q$, where $p$ is “$f$ is differentiable”, and $q$ is “$f$ is continuous”.

(c) $p \rightarrow q$, where $p$ is “grass will grow”, and $q$ is “enough moisture is available”.

(d) $(p \lor q) \rightarrow \neg r$, where $p$ is “taxes are increased”, $q$ is “government spending decreases”, and $r$ is “inflation will occur this year”.

Problem 3:

(a) This is a contradiction. Indeed, let

$$P = (a \rightarrow b) \rightarrow \neg(b \rightarrow a)$$

and let

$$Q = a \leftrightarrow b.$$

To show that $P \leftrightarrow Q$ is a contradiction, it is enough to show that

$$P \iff \neg Q.$$
We first note that
\[
\neg Q \iff \neg [(-a \lor b) \land (-b \lor a)] \\
\iff \neg(-a \lor b) \lor \neg(-b \lor a) \\
\iff (a \land \neg b) \lor (b \land \neg a).
\]

Now,
\[
P \iff (-a \lor b) \rightarrow \neg(-b \lor a) \\
\iff (-a \lor b) \rightarrow (b \land \neg a) \\
\iff \neg(-a \lor b) \lor (b \land \neg a) \\
\iff (a \land \neg b) \lor (b \land \neg a).
\]

Thus \( P \iff \neg Q \). \( \square \)

(b) This one is neither a tautology nor a contradiction. If we let both \( a \) and \( b \) to be true, we get that the overall statement is true; while if we let \( a \) to be true and \( b \) to be false, then we obtain that the statement is false. Indeed it can be shown that
\[
[(a \rightarrow b) \rightarrow b] \rightarrow b \iff a \rightarrow b
\]
which obviously is neither a tautology nor a contradiction.

(c) This is a contradiction. This is shown as follows.
\[
b \land \neg(a \lor b) \iff b \land (\neg a \land \neg b) \\
\iff (b \land \neg b) \land \neg a \\
\iff F \land \neg a \\
\iff F,
\]
where \( F \) denotes a contradiction. \( \square \)

(d) This is a tautology since
\[
(a \rightarrow (b \land \neg b)) \rightarrow \neg a \iff (a \rightarrow F) \rightarrow \neg a \\
\iff \neg(a \lor F) \rightarrow \neg a \\
\iff \neg a \rightarrow \neg a \\
\iff T,
\]
where \( T \) denotes a contradiction. \( \square \)
Problem 4:

(a) \( \neg(p \leftrightarrow q) \iff \neg[\neg(p \lor q) \land (p \lor \neg q)] \iff (p \land \neg q) \lor (\neg p \land q) \).

(b) \( \neg[p \rightarrow (q \rightarrow r)] \iff \neg[\neg p \lor (\neg q \lor r)] \iff p \land q \land \neg r \).

(c) \( \neg[(p \land (q \rightarrow r)) \lor (\neg q \land p)] \iff \neg[p \lor \neg (\neg q \lor r)] \land (q \lor \neg p) \iff \neg p \lor [(q \land \neg r) \land (q \land \neg r)] \iff \neg p \lor [(q \land \neg r) \land (q \land \neg r) \land \neg p] \iff \neg p \lor (q \land \neg r), \) where the fourth and last steps follow from the absorption laws.

Problem 5:

Let \( a \) : “PM is reelected,” \( b \) : “PM provides substantial tax cuts,” and \( c \) : “budget surplus is larger than $10 billion.

Then the argument becomes:

\[ S \implies \neg a, \]

where

\[ S = (b \implies a) \land (b \implies c) \land \neg c. \]

The argument is valid iff \( S \implies \neg a \) is a tautology. Let us use the “short-cut” approach (instead of a truth table) by checking if there exits a truth assignment to the triplet \((a, b, c)\) that will make \( S \implies \neg a \) false. If the latter is true, we must have that \( S \) is T (true) and that \( \neg a \) is F (false). Thus

\[
\begin{aligned}
b \implies a & \; \text{is T} \\
b \implies c & \; \text{is T} \\
\neg c & \; \text{is T} \\
a & \; \text{is T}
\end{aligned}
\]

thus

\[
\begin{aligned}
a & \; \text{is T} \\
c & \; \text{is F} \\
b \implies a & \; \text{is T} \\
b \implies c & \; \text{is T}
\end{aligned}
\]
thus
\[
\begin{cases}
  a \text{ is } T \\
  c \text{ is } F \\
  b \text{ is } F
\end{cases}
\]
Indeed, we can have a possible truth assignment on \((a, b, c)\) for which \(S \rightarrow \neg a\) is false. Thus the argument is invalid.