Problem: Find the equations of planes that just touch the sphere \((x-2)^2 + (y-3)^2 + (z-3)^2 = 16\) and are parallel to
(a) The \(xy\) plane
(b) The \(yz\) plane
(c) The \(xz\) plane

Solution. From the equation of the sphere, it is clear that the center of the sphere has co-ordinates \((2, 3, 3)\) and that the sphere has radius 4.

(a) A plane which is parallel to the \(xy\) plane is given by \(z = c\) for some constant \(c\). Drawing the sphere in 3-space, we see that planes of the form \(z = c\) which just touch the sphere, must be 4 above and 4 below the center. Thus the required planes are \(z = 7\) and \(z = 1\), touching the sphere at the points \((2, 3, 7)\) and \((2, 3, 1)\) respectively.

An alternative method is to consider the intersection of the plane \(z = c\) with the sphere, given by the equation
\[
(x-2)^2 + (y-3)^2 + (c-3)^2 = 16.
\]

We want to find the value of \(c\) for which this intersection is a single point.

If \((c-3)^2 > 16\), \((*)\) has no solution since \((x-2)^2\) and \((y-3)^2\) are non-negative. Hence the intersection of the plane with the sphere is empty. If \((c-3)^2 < 16\), \((*)\) describes a circle of non-zero radius. If \((c-3)^2 = 16\), \((*)\) reduces to \((x-2)^2 + (y-3)^2 = 0\), whose solution is simply the point \((2, 3)\). This is the case we are looking for and hence \(c\) can be either 7 or \(-1\).

(b) Can be solved similarly to get the planes \(x = -2\) and \(x = 6\).

(c) Can be solved similarly to get the planes \(y = -1\) and \(y = 7\).