Q 1: A force field is described by $\vec{F} = 2xy\hat{i} + (x^2 + 2yz)\hat{j} + y^2\hat{k}$.

(a) Is $\vec{F}$ conservative? Why or why not?
(b) A particle moves from the point $c(0)$ to the point $c(\pi/6)$ along the helix parametrized by $c(t) = (\cos t, \sin t, t)$. Find the work done by $\vec{F}$ on the particle.

Q 2. Let $S$ be the part of the paraboloid $z = -(x^2 + y^2) + 2$ lying above the rectangle $0 \leq x \leq 1, 0 \leq y \leq 1$ in the $xy$-plane (with upwards orientation). Let $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$. Calculate the flux of $\vec{F}$ through $S$.

Q 3. Let $f(x, y, z) = x^2 + y^2 + z^2$. Calculate the flux of $\vec{F} = xy^2\hat{i} + z^2y\hat{j} + x^2z\hat{k}$ through the level surface $f(x, y, z) = 4$.

Q 4. Evaluate

(a) $\int_0^1 \int_{e^y}^e \frac{x}{\log x} \, dx \, dy$

(b) $\int_0^1 \int_{e^y}^e \frac{\log x}{x} \, dx \, dy$

Q 5. Let $C$ be the curve with parametric equations $x = 2 + t^2, y = 3 - 2t, z = 5 - t^2$.

(a) Find the velocity vector and the acceleration vector of the curve.
(b) Find the tangent line to the curve at $(3, 5, 4)$.

Q 6. Consider the function

$$f(x, y) = \begin{cases} 
\frac{xy^2}{x^2 + y^4}, & (x, y) \neq (0, 0) \\
0, & (x, y) = (0, 0) 
\end{cases}$$

(a) Is $f$ continuous at $(0, 0)$?
(b) Use the definition of differentiability to check whether $f$ is differentiable at $(0, 0)$.

Q 7. Find $\partial z/\partial u$ and $\partial z/\partial v$ for

$$z = xe^{-y} + ye^{-x}, \quad x = u \sin v, \quad y = v \cos u.$$
Q 8. A pile of hay is approximately in the shape of $0 \leq z \leq 2 - x^2 - y^2$, where $x, y, z$ are in meters. At height $z$, the density of the hay is $\delta = (2 - z)$ kg/m3. Find the mass of hay in the pile.

Q 9. Find the volume and surface area of the parallelepiped made by the vectors $\vec{a} = (1, 0, 2), \vec{b} = (2, 1, 1), \vec{c} = (1, 2, 0)$.

Q 10. Find the area enclosed by the circle $x^2 + y^2 = 4$ using Green’s theorem.