1. Note that \( \frac{dy}{dt} = 0 \) if and only if \( y = -3 \). Therefore, the constant function \( y(t) = -3 \) for all \( t \) is the only equilibrium solution.

2. Note that \( \frac{dy}{dt} = 0 \) for all \( t \) only if \( y^2 - 2 = 0 \). Therefore, the only equilibrium solutions are \( y(t) = -\sqrt{2} \) for all \( t \) and \( y(t) = +\sqrt{2} \) for all \( t \).

4. (a) The equilibrium solutions correspond to the values of \( P \) for which \( \frac{dP}{dt} = 0 \) for all \( t \). For this equation, \( \frac{dP}{dt} = 0 \) for all \( t \) if \( P = 0 \), \( P = 50 \), or \( P = 200 \).

   (b) The population is increasing if \( \frac{dP}{dt} > 0 \). That is, \( P < 0 \) or \( 50 < P < 200 \). Note, \( P < 0 \) might be considered “nonphysical” for a population model.

   (c) The population is decreasing if \( \frac{dP}{dt} < 0 \). That is, \( 0 < P < 50 \) or \( P > 200 \).

5. In order to answer the question, we first need to analyze the sign of the polynomial \( y^3 - y^2 - 12y \). Factoring, we obtain

\[
y^3 - y^2 - 12y = y(y^2 - y - 12) = y(y - 4)(y + 3).
\]

(a) The equilibrium solutions correspond to the values of \( y \) for which \( \frac{dy}{dt} = 0 \) for all \( t \). For this equation, \( \frac{dy}{dt} = 0 \) for all \( t \) if \( y = -3 \), \( y = 0 \), or \( y = 4 \).

(b) The solution \( y(t) \) is increasing if \( \frac{dy}{dt} > 0 \). That is, \( -3 < y < 0 \) or \( y > 4 \).

(c) The solution \( y(t) \) is decreasing if \( \frac{dy}{dt} < 0 \). That is, \( y < -3 \) or \( 0 < y < 4 \).

6. (a) The rate of change of the amount of radioactive material is \( \frac{dr}{dt} \). This rate is proportional to the amount \( r \) of material present at time \( t \). With \( -\lambda \) as the proportionality constant, we obtain the differential equation

\[
\frac{dr}{dt} = -\lambda r.
\]

Note that the minus sign (along with the assumption that \( \lambda \) is positive) means that the material decays.

(b) The only additional assumption is the initial condition \( r(0) = r_0 \). Consequently, the corresponding initial-value problem is

\[
\frac{dr}{dt} = -\lambda r, \quad r(0) = r_0.
\]
1. \((a)\) Let’s check Bob’s solution first. Since \(dy/dt = 1\) and
\[
\frac{y(t) + 1}{t + 1} = \frac{t + 1}{t + 1} = 1,
\]
Bob’s answer is correct.

Now let’s check Glen’s solution. Since \(dy/dt = 2\) and
\[
\frac{y(t) + 1}{t + 1} = \frac{2t + 2}{t + 1} = 2,
\]
Glen’s solution is also correct.

Finally let’s check Paul’s solution. We have \(dy/dt = 2t\) on one hand and
\[
\frac{y(t) + 1}{t + 1} = \frac{t^2 - 1}{t + 1} = t - 1
\]
on the other. Paul is wrong.

\((b)\) At first glance, they should have seen the equilibrium solution \(y(t) = -1\) for all \(t\) because \(dy/dt = 0\) for any constant function and \(y = -1\) implies that
\[
\frac{y + 1}{t + 1} = 0
\]

independent of \(t\).

Strictly speaking the differential equation is not defined for \(t = -1\), and hence the solutions are not defined for \(t = -1\).

5. \((a)\) This equation is separable. (It is nonlinear and nonautonomous as well.)

\((b)\) We separate variables and integrate to obtain
\[
\int \frac{1}{y^2} \, dy = \int t^2 \, dt
\]
\[\frac{-1}{y} = \frac{t^3}{3} + c\]
\[y(t) = \frac{-1}{(t^3/3) + c},\]
where \(c\) is any real number. This function can also be written in the form
\[y(t) = \frac{-3}{t^3 + k}\]
where \(k\) is any constant. The constant function \(y(t) = 0\) for all \(t\) is also a solution of this equation. It is the equilibrium solution at \(y = 0\).
7. We separate variables and integrate to obtain

\[ \int \frac{dy}{2y + 1} = \int dt. \]

We get

\[ \frac{1}{2} \ln |2y + 1| = t + c \]

\[ |2y + 1| = e^{2t}, \]

where \( c_1 = e^{2c} \). As in Exercise 22, we can drop the absolute value signs by replacing \( \pm c_1 \) with a new constant \( k_1 \). Hence, we have

\[ 2y + 1 = k_1 e^{2t} \]

\[ y = \frac{1}{2} \left( k_1 e^{2t} - 1 \right), \]

and letting \( k = k_1/2 \), \( y(t) = k e^{2t} - 1/2 \). Note that, for \( k = 0 \), we get the equilibrium solution.

10. We separate variables and obtain

\[ \int \frac{dx}{1 + x^2} = \int 1 \, dt. \]

Integrating both sides, we get

\[ \arctan x = t + c, \]

where \( c \) is a constant. Hence, the general solution is

\[ x(t) = \tan(t + c). \]

14. Separating variables and integrating, we obtain

\[ \int y^{-1/3} \, dy = \int t \, dt \]

\[ \frac{3}{2} y^{2/3} = \frac{t^2}{2} + k \]

\[ y^{2/3} = \frac{t^2}{3} + c, \]

where \( c = 2k/3 \). Hence,

\[ y(t) = \pm \left( \frac{t^2}{3} + c \right)^{3/2}. \]

Note that this form does not include the equilibrium solution \( y = 0 \).
18. Separating variables and integrating, we have
\[ \int (1 + 3y^2) \, dy = \int 4t \, dt \]
\[ y + y^3 = 2t^2 + c. \]

To express \( y \) as a function of \( t \), we must solve a cubic. The equation for the roots of a cubic can be found in old algebra books or by asking a computer algebra program. But we do not learn a lot from the result.

19. The equation can be written in the form
\[ \frac{dv}{dt} = (v + 1)(t^2 - 2), \]
and we note that \( v(t) = -1 \) for all \( t \) is an equilibrium solution. Separating variables and integrating, we obtain
\[ \int \frac{dv}{v + 1} = \int t^2 - 2 \, dt \]
\[ \ln |v + 1| = \frac{t^3}{3} - 2t + c, \]
where \( c \) is any constant. Thus,
\[ |v + 1| = c_1 e^{-2t + t^3/3}, \]
where \( c_1 = e^c \). We can dispose of the absolute value signs by allowing the constant \( c_1 \) to be any real number. In other words,
\[ v(t) = -1 + ke^{-2t + t^3/3}, \]
where \( k = \pm c_1 \). Note that, if \( k = 0 \), we get the equilibrium solution.
25. Separating variables and integrating, we have
\[
\int \frac{1}{x} \, dx = -\int t \, dt
\]
\[
\ln |x| = -\frac{t^2}{2} + c
\]
\[
|x| = k_1 e^{-t^2/2},
\]
where \( k_1 = e^c \). We can eliminate the absolute value signs by allowing the constant \( k_1 \) to be either positive or negative. Thus, the general solution is
\[
x(t) = k e^{-t^2/2}
\]
where \( k = \pm k_1 \). Using the initial condition to solve for \( k \), we have
\[
\frac{1}{\sqrt{\pi}} = x(0) = ke^0 = k.
\]
Therefore,
\[
x(t) = \frac{e^{-t^2/2}}{\sqrt{\pi}}.
\]

27. Separating variables and integrating, we obtain
\[
\int \frac{dy}{y^2} = -\int dt
\]
\[
-\frac{1}{y} = -t + c.
\]
So we get
\[
y = \frac{1}{t - c}.
\]
Now we need to find the constant \( c \) so that \( y(0) = 1/2 \). To do this we solve
\[
\frac{1}{2} = \frac{1}{0 - c}
\]
and get \( c = -2 \). The solution of the initial-value problem is
\[
y(t) = \frac{1}{t + 2}.
\]
32. First we find the general solution by writing the differential equation as

\[ \frac{dy}{dt} = (t + 2)y^2, \]

separating variables, and integrating. We have

\[
\int \frac{1}{y^2} \, dy = \int (t + 2) \, dt
\]

\[-\frac{1}{y} = \frac{t^2}{2} + 2t + c
\]

\[= \frac{t^2 + 4t + c_1}{2},\]

where \(c_1 = 2c\). Inverting and multiplying by \(-1\) produces

\[ y(t) = \frac{-2}{t^2 + 4t + c_1}. \]

Setting

\[ 1 = y(0) = \frac{-2}{c_1} \]

and solving for \(c_1\), we obtain \(c_1 = -2\). So

\[ y(t) = \frac{-2}{t^2 + 4t - 2}. \]
35. We separate variables to obtain

\[
\int \frac{dy}{1 + y^2} = \int t \, dt
\]

\[
\arctan y = \frac{t^2}{2} + c,
\]

where \( c \) is a constant. Hence the general solution is

\[
y(t) = \tan \left( \frac{t^2}{2} + c \right).
\]

Next we find \( c \) so that \( y(0) = 1 \). Solving

\[
1 = \tan \left( \frac{0^2}{2} + c \right)
\]

yields \( c = \pi/4 \), and the solution to the initial-value problem is

\[
y(t) = \tan \left( \frac{t^2}{2} + \frac{\pi}{4} \right).
\]