5. \( \mathbf{Y} = \begin{pmatrix} x \\ y \end{pmatrix} \), \( \frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \mathbf{Y} \)

8. \( \frac{dx}{dt} = -3x + 2\pi y \)
\( \frac{dy}{dt} = 4x - y \)

14. (a) If \( a = 0 \), then \( \det \mathbf{A} = ad - bc = bc \). Thus both \( b \) and \( c \) are nonzero if \( \det \mathbf{A} \neq 0 \).

(b) Equilibrium points \((x_0, y_0)\) are solutions of the simultaneous system of linear equations

\[
\begin{align*}
a x_0 + b y_0 &= 0 \\
c x_0 + d y_0 &= 0.
\end{align*}
\]

If \( a = 0 \), the first equation reduces to \( b y_0 = 0 \), and since \( b \neq 0 \), \( y_0 = 0 \). In this case, the second equation reduces to \( c x_0 = 0 \), so \( x_0 = 0 \) as well. Therefore, \((x_0, y_0) = (0, 0)\) is the only equilibrium point for the system.
15. The vector field at a point \((x_0, y_0)\) is \((ax_0 + by_0, cx_0 + dy_0)\), so in order for a point to be an equilibrium point, it must be a solution to the system of simultaneous linear equations

\[
\begin{align*}
ax_0 + by_0 &= 0 \\
cx_0 + dy_0 &= 0.
\end{align*}
\]

If \(a \neq 0\), we know that the first equation is satisfied if and only if

\[x_0 = -\frac{b}{a} y_0.\]

Now we see that any point that lies on this line \(x_0 = (-b/a)y_0\) also satisfies the second linear equation \(cx_0 + dy_0 = 0\). In fact, if we substitute a point of this form into the second component of the vector field, we have

\[
cx_0 + dy_0 = c \left( -\frac{b}{a} \right) y_0 + dy_0
\]

\[
= \left( -\frac{bc}{a} + d \right) y_0
\]

\[
= \left( -\frac{ad - bc}{a} \right) y_0
\]

\[
= \frac{\det A}{a} y_0
\]

\[= 0,
\]

since we are assuming that \(\det A = 0\). Hence, the line \(x_0 = (-b/a)y_0\) consists entirely of equilibrium points.

If \(a = 0\) and \(b \neq 0\), then the determinant condition \(\det A = ad - bc = 0\) implies that \(c = 0\). Consequently, the vector field at the point \((x_0, y_0)\) is \((by_0, dy_0)\). Since \(b \neq 0\), we see that we get equilibrium points if and only if \(y_0 = 0\). In other words, the set of equilibrium points is exactly the \(x\)-axis.

Finally, if \(a = b = 0\), then the vector field at the point \((x_0, y_0)\) is \((0, cx_0 + dy_0)\). In this case, we see that a point \((x_0, y_0)\) is an equilibrium point if and only if \(cx_0 + dy_0 = 0\). Since at least one of \(c\) or \(d\) is nonzero, the set of points \((x_0, y_0)\) that satisfy \(cx_0 + dy_0 = 0\) is precisely a line through the origin.
30. (a) This holds in all dimensions. In two dimensions the computation is
\[
    \mathbf{A} k \mathbf{y} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} k \begin{pmatrix} x \\ y \end{pmatrix}
\]
\[
    = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} kx \\ ky \end{pmatrix}
\]
\[
    = \begin{pmatrix} akx + bky \\ cky + dky \end{pmatrix}
\]
\[
    = k \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix} = k \mathbf{A} \mathbf{y}.
\]

(b) To verify the first half of the Linearity Principle, we suppose that \(\mathbf{Y}_1(t) = (x_1(t), y_1(t))\) is a solution to the system
\[
    \frac{dx}{dt} = ax + by
\]
\[
    \frac{dy}{dt} = cx + dy
\]
and that \(k\) is an any constant. In order to verify that the function \(\mathbf{Y}_2(t) = k\mathbf{Y}_1(t)\) is also a solution, we need to substitute \(\mathbf{Y}_2(t)\) into both sides of the differential equation and check for equality. In other words, after we write \(\mathbf{Y}_2(t)\) in scalar notation as \(\mathbf{Y}_2(t) = (x_2(t), y_2(t))\), we must show that
\[
    \frac{dx_2}{dt} = ax_2 + by_2
\]
\[
    \frac{dy_2}{dt} = cx_2 + dy_2
\]
given that we know that
\[
    \frac{dx_1}{dt} = ax_1 + by_1
\]
\[
    \frac{dy_1}{dt} = cx_1 + dy_1.
\]
Since \( x_2(t) = kx_1(t) \) and \( y_2(t) = ky_1(t) \), we can multiply both sides of

\[
\frac{dx_1}{dt} = ax_1 + by_1 \\
\frac{dy_1}{dt} = cx_1 + dy_1.
\]

by \( k \) to obtain

\[
k \frac{dx_1}{dt} = k(ax_1 + by_1) \\
k \frac{dy_1}{dt} = k(cx_1 + dy_1).
\]

However, using standard algebraic properties and the rules of differentiation, this system is equivalent to

\[
\frac{d(kx_1)}{dt} = a(kx_1) + b(ky_1) \\
\frac{d(ky_1)}{dt} = c(kx_1) + d(ky_1),
\]

which is the same as the desired equality

\[
\frac{dx_2}{dt} = ax_2 + by_2 \\
\frac{dy_2}{dt} = cx_2 + dy_2.
\]

To verify the second half of the Linearity Principle, we suppose that \( Y_1(t) = (x_1(t), y_1(t)) \) and \( Y_2(t) = (x_2(t), y_2(t)) \) are solutions to the system

\[
\frac{dx}{dt} = ax + by \\
\frac{dy}{dt} = cx + dy.
\]
To verify that the function $Y_3(t) = Y_1(t) + Y_2(t)$ is also a solution, we need to substitute $Y_3(t)$ into both sides of the differential equation and check for equality. In other words, after we write $Y_3(t)$ in scalar notation as $Y_3(t) = (x_3(t), y_3(t))$, we must show that

$$\frac{dx_3}{dt} = ax_3 + by_3$$
$$\frac{dy_3}{dt} = cx_3 + dy_3$$

given that we know that

$$\frac{dx_1}{dt} = ax_1 + by_1$$
$$\frac{dy_1}{dt} = cx_1 + dy_1$$

and

$$\frac{dx_2}{dt} = ax_2 + by_2$$
$$\frac{dy_2}{dt} = cx_2 + dy_2.$$ 

Adding the two given systems together yields the system

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = ax_1 + by_1 + ax_2 + by_2$$
$$\frac{dy_1}{dt} + \frac{dy_2}{dt} = cx_1 + dy_1 + cx_2 + dy_2,$$

which can be rewritten as

$$\frac{d(x_1 + x_2)}{dt} = a(x_1 + x_2) + b(y_1 + y_2)$$
$$\frac{d(y_1 + y_2)}{dt} = c(x_1 + x_2) + d(y_1 + y_2).$$

But this last system of equalities is the desired equality that indicates that $Y_3(t)$ is also a solution.
11. The eigenvalues are the roots of the characteristic polynomial, so they are solutions of

\[ (-2 - \lambda)(1 - \lambda) - 4 = \lambda^2 + \lambda - 6 = 0. \]

Hence, \( \lambda_1 = 2 \) and \( \lambda_2 = -3 \) are the eigenvalues.

To find the eigenvectors for the eigenvalue \( \lambda_1 = 2 \), we solve

\[
\begin{align*}
-2x_1 - 2y_1 &= 2x_1 \\
-2x_1 + y_1 &= 2y_1,
\end{align*}
\]

so \( y_1 = -2x_1 \) is the line of eigenvectors. In particular, \( (1, -2) \) is an eigenvector for \( \lambda_1 = 2 \).

Similarly, the line of eigenvectors for \( \lambda_2 = -3 \) is given by \( x_1 = 2y_1 \). In particular, \( (2, 1) \) is an eigenvector for \( \lambda_2 = -3 \).

Given the eigenvalues and these eigenvectors, we have the two linearly independent solutions

\[
Y_1(t) = e^{2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad \text{and} \quad Y_2(t) = e^{-3t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}.
\]

The general solution is

\[
Y(t) = k_1 e^{2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + k_2 e^{-3t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}.
\]

(a) Given the initial condition \( Y(0) = (1, 0) \), we must solve

\[
\begin{pmatrix} 1 \\ 0 \end{pmatrix} = Y(0) = k_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + k_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix}
\]

for \( k_1 \) and \( k_2 \). This vector equation is equivalent to the two scalar equations

\[
\begin{align*}
    k_1 + 2k_2 &= 1 \\
    -2k_1 + k_2 &= 0.
\end{align*}
\]

Solving these equations, we obtain \( k_1 = 1/5 \) and \( k_2 = 2/5 \). Thus, the particular solution is

\[
Y(t) = \frac{1}{5} e^{2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \frac{2}{5} e^{-3t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}.
\]
(b) Given the initial condition $\mathbf{Y}(0) = (0, 1)$ we must solve

$$
\begin{pmatrix}
0 \\
1
\end{pmatrix}
= \mathbf{Y}(0) = k_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + k_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix}
$$

for $k_1$ and $k_2$. This vector equation is equivalent to the two scalar equations

$$
\begin{cases}
k_1 + 2k_2 = 0 \\
-2k_1 + k_2 = 1.
\end{cases}
$$

Solving these equations, we obtain $k_1 = -2/5$ and $k_2 = 1/5$. Thus, the particular solution is

$$
\mathbf{Y}(t) = -\frac{2}{5}e^{2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \frac{1}{5}e^{-3t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}.
$$

(e) The initial condition $\mathbf{Y}(0) = (1, -2)$ is an eigenvector for the eigenvalue $\lambda_1 = 2$. Hence, the solution with this initial condition is

$$
\mathbf{Y}(t) = e^{2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}.
$$
12. The characteristic polynomial is

\[(3 - \lambda)(-2 - \lambda) = 0,\]

and therefore the eigenvalues are \(\lambda_1 = 3\) and \(\lambda_2 = -2\).

To obtain the eigenvectors \((x_1, y_1)\) for the eigenvalue \(\lambda_1 = 3\), we solve the system of equations

\[
\begin{align*}
3x_1 &= 3x_1 \\
x_1 - 2y_1 &= 3y_1
\end{align*}
\]

and obtain

\[5y_1 = x_1.\]

Therefore, an eigenvector for the eigenvalue \(\lambda_1 = 3\) is \(\mathbf{v}_1 = (5, 1)\).

Using the same procedure, we obtain the eigenvector \(\mathbf{v}_2 = (0, 1)\) for \(\lambda_2 = -2\).

The general solution to this linear system is therefore

\[\mathbf{y}(t) = k_1 e^{3t} \begin{pmatrix} 5 \\ 1 \end{pmatrix} + k_2 e^{-2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}.\]

(a) We have \(\mathbf{y}(0) = (1, 0)\), so we must find \(k_1\) and \(k_2\) so that

\[\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \mathbf{y}(0) = k_1 \begin{pmatrix} 5 \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}.\]

This vector equation is equivalent to the simultaneous system of linear equations

\[
\begin{align*}
5k_1 &= 1 \\
k_1 + k_2 &= 0.
\end{align*}
\]

Solving these equations, we obtain \(k_1 = 1/5\) and \(k_2 = -1/5\). Thus, the particular solution is

\[\mathbf{y}(t) = \frac{1}{5} e^{3t} \begin{pmatrix} 5 \\ 1 \end{pmatrix} - \frac{1}{5} e^{-2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}.\]
(b) We have \( Y(0) = (0, 1) \). Since this initial condition is an eigenvector associated to the \( \lambda = -2 \) eigenvalue, we do not need to do any additional calculation. The desired solution to the initial-value problem is

\[
Y(t) = e^{-2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
\]

(c) We have \( Y(0) = (2, 2) \), so we must find \( k_1 \) and \( k_2 \) so that

\[
\begin{pmatrix} 2 \\ 2 \end{pmatrix} = Y(0) = k_1 \begin{pmatrix} 5 \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
\]

This vector equation is equivalent to the simultaneous system of linear equations

\[
\begin{align*}
5k_1 &= 2 \\
k_1 + k_2 &= 2.
\end{align*}
\]

Solving these equations, we obtain \( k_1 = 2/5 \) and \( k_2 = 8/5 \). Thus, the particular solution is

\[
Y(t) = \frac{2}{5} e^{3t} \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \frac{8}{5} e^{-2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
\]

15. Given any vector \( Y_0 = (x_0, y_0) \), we have

\[
AY_0 = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} ax_0 \\ ay_0 \end{pmatrix} = a \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = aY_0.
\]

Therefore, every nonzero vector is an eigenvector associated to the eigenvalue \( a \).
16. The characteristic polynomial of $A$ is

$$(a - \lambda)(d - \lambda) = 0,$$

and thus the eigenvalues of $A$ are $\lambda_1 = a$ and $\lambda_2 = d$.

To find the eigenvectors $\mathbf{V}_1 = (x_1, y_1)$ associated to $\lambda_1 = a$, we need to solve the equation

$$A\mathbf{V}_1 = a\mathbf{V}_1$$

for all possible vectors $\mathbf{V}_1$. Rewritten in terms of components, this equation is equivalent to

$$\begin{cases} 
x_1 + by_1 = ax_1 \\
dy_1 = ay_1.
\end{cases}$$

Since $a \neq d$, the second equation implies that $y_1 = 0$. If so, then the first equation is satisfied for all $x_1$. In other words, the eigenvectors $\mathbf{V}_1$ associated to the eigenvalue $a$ are the vectors of the form $(x_1, 0)$.

To find the eigenvectors $\mathbf{V}_2 = (x_2, y_2)$ associated to $\lambda_2 = d$, we need to solve the equation

$$A\mathbf{V}_2 = d\mathbf{V}_2$$

for all possible vectors $\mathbf{V}_2$. Rewritten in terms of components, this equation is equivalent to

$$\begin{cases} 
x_2 + by_2 = dx_2 \\
dy_2 = dy_2.
\end{cases}$$

The second equation always holds, so the eigenvectors $\mathbf{V}_2$ are those vectors that satisfy the equation $ax_2 + by_2 = dx_2$, which can be rewritten as

$$by_2 = (d - a)x_2.$$

These vectors form a line through the origin of slope $(d - a)/b.$
9. As we computed in Exercise 11 of Section 3.2, the eigenvalues are $\lambda_1 = 2$ and $\lambda_2 = -3$. The eigenvectors $(x_1, y_1)$ for the eigenvalue $\lambda_1 = 2$ satisfy $y_1 = -2x_1$, and the eigenvectors for the eigenvalue $\lambda_2 = -3$ satisfy $x_1 = 2y_1$. The equilibrium point at the origin is a saddle. The solution curves in the phase plane for the initial conditions $(1, 0)$, $(0, 1)$, and $(1, -2)$ are shown in the figure on the right.

(a) The solution with initial condition $(1, 0)$ is asymptotic to the line $y = -2x$ in the fourth quadrant as $t \to \infty$ and to the line $x = 2y$ in the first quadrant as $t \to -\infty$.

(b) The solution curve with initial condition $(1, 0)$ is asymptotic to the line $y = -2x$ in the second quadrant as $t \to \infty$ and to the line $x = 2y$ in the first quadrant as $t \to -\infty$. 
(c) The solution curve with initial condition \((1, -2)\) is on the line of eigenvectors for the eigenvalue \(\lambda_1 = 2\). Hence, this solution curve stays on the line \(y = -2x\). It approaches the origin as \(t \to -\infty\), and it tends to \(\infty\) in the fourth quadrant as \(t \to \infty\).
10. As we computed in Exercise 12 of Section 3.2, the eigenvalues are \( \lambda_1 = 3 \) and \( \lambda_2 = -2 \). The eigenvectors \((x_1, y_1)\) for the eigenvalue \( \lambda_1 = 3 \) satisfy \( y_1 = x_1/5 \), and the eigenvectors \((x_2, y_2)\) for the eigenvalue \( \lambda_2 = -2 \) satisfy \( x_2 = 0 \). The equilibrium point at the origin is a saddle. Therefore, the solution curves in the phase plane for the initial conditions \((1, 0)\), \((0, 1)\), and \((2, 2)\) are shown in the figure on the right.

(a) The solution curve with initial condition \((1, 0)\) is asymptotic to the negative \( y \)-axis as \( t \to -\infty \) and is asymptotic to the line \( y = x/5 \) in the first quadrant as \( t \to \infty \).

(b) The solution curve with initial condition \((0, 1)\) lies entirely on the positive \( y \)-axis, and \( y(t) \to 0 \) in an exponential fashion as \( t \to \infty \).
(e) The solution curve with initial condition \((2, 2)\) lies entirely in the first quadrant. It is asymptotic to the positive \(y\)-axis as \(t \to -\infty\) and asymptotic to the line \(y = x/5\) as \(t \to \infty\).

![Graph](image)

1. Using Euler’s formula, we can write the complex-valued solution \(Y_c(t)\) as

\[
Y_c(t) = e^{(1 + 3i)t} \begin{pmatrix} 2 + i \\ 1 \end{pmatrix}
\]

\[
= e^t e^{3it} \begin{pmatrix} 2 + i \\ 1 \end{pmatrix}
\]

\[
= e^t (\cos 3t + i \sin 3t) \begin{pmatrix} 2 + i \\ 1 \end{pmatrix}
\]

\[
= e^t \begin{pmatrix} 2 \cos 3t - \sin 3t \\ \cos 3t \end{pmatrix} + i e^t \begin{pmatrix} 2 \sin 3t + \cos 3t \\ \sin 3t \end{pmatrix}.
\]

Hence, we have

\[
Y_{re}(t) = e^t \begin{pmatrix} 2 \cos 3t - \sin 3t \\ \cos 3t \end{pmatrix} \quad \text{and} \quad Y_{im}(t) = e^t \begin{pmatrix} \cos 3t + 2 \sin 3t \\ \sin 3t \end{pmatrix}.
\]

The general solution is

\[
Y(t) = k_1 e^t \begin{pmatrix} 2 \cos 3t - \sin 3t \\ \cos 3t \end{pmatrix} + k_2 e^t \begin{pmatrix} \cos 3t + 2 \sin 3t \\ \sin 3t \end{pmatrix}.
\]
3. (a) The characteristic equation is

\[ (-\lambda)^2 + 4 = \lambda^2 + 4 = 0, \]

and the eigenvalues are \( \lambda = \pm 2i \).

(b) Since the real part of the eigenvalues are 0, the origin is a center.

(c) Since \( \lambda = \pm 2i \), the natural period is \( 2\pi/2 = \pi \), and the natural frequency is \( 1/\pi \).

(d) At \( (1, 0) \), the tangent vector is \( (-2, 0) \). Therefore, the direction of oscillation is clockwise.

(e) According to the phase plane, \( x(t) \) and \( y(t) \) are periodic with period \( \pi \). At the initial condition \( (1, 0) \), both \( x(t) \) and \( y(t) \) are initially decreasing.
4. (a) The characteristic equation is

\[(2 - \lambda)(6 - \lambda) + 8 = \lambda^2 - 8\lambda + 20,\]

and the eigenvalues are \(\lambda = 4 \pm 2i\).

(b) Since the real part of the eigenvalues is positive, the origin is a spiral source.

(c) Since \(\lambda = 4 \pm 2i\), the natural period is \(2\pi/2 = \pi\), and the natural frequency is \(1/\pi\).

(d) At the point \((1, 0)\), the tangent vector is \((2, -4)\). Therefore, the solution curves spiral around the origin in a clockwise fashion.

(e) Since \(dY/dt = (4, 2)\) at \(Y_0 = (1, 1)\), both \(x(t)\) and \(y(t)\) increase initially. The distance between successive zeros is \(\pi\), and the amplitudes of both \(x(t)\) and \(y(t)\) are increasing.
9. (a) According to Exercise 3, \( \lambda = \pm 2i \). The eigenvectors \((x, y)\) associated to eigenvalue \( \lambda = 2i \) must satisfy the equation \( 2y = 2ix \), which is equivalent to \( y = ix \). One such eigenvector is \((1, i)\), and thus we have the complex solution
\[
Y(t) = e^{2it} \begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} \cos 2t \\ -\sin 2t \end{pmatrix} + i \begin{pmatrix} \sin 2t \\ \cos 2t \end{pmatrix}.
\]
Taking real and imaginary parts, we obtain the general solution
\[
Y(t) = k_1 \begin{pmatrix} \cos 2t \\ -\sin 2t \end{pmatrix} + k_2 \begin{pmatrix} \sin 2t \\ \cos 2t \end{pmatrix}.
\]
(b) From the initial condition, we obtain
\[
k_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + k_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix},
\]
and therefore, \( k_1 = 1 \) and \( k_2 = 0 \). Consequently, the solution with the initial condition \((1, 0)\) is
\[
Y(t) = \begin{pmatrix} \cos 2t \\ -\sin 2t \end{pmatrix}.
\]
(c)
10. (a) According to Exercise 4, the eigenvalues are $\lambda = 4 \pm 2i$. The eigenvectors $(x, y)$ associated to the eigenvalue $4 + 2i$ must satisfy the equation $y = (1 + i)x$. Hence, using the eigenvector $(1, 1 + i)$, we obtain the complex-valued solution

$$Y(t) = e^{(4+2i)t} \begin{pmatrix} 1 \\ 1 + i \end{pmatrix} = e^{4t} \begin{pmatrix} \cos 2t \\ \cos 2t - \sin 2t \end{pmatrix} + ie^{4t} \begin{pmatrix} \sin 2t \\ \cos 2t + \sin 2t \end{pmatrix}.$$ 

From the real and imaginary parts of $Y(t)$, we obtain the general solution

$$Y(t) = k_1 e^{4t} \begin{pmatrix} \cos 2t \\ \cos 2t - \sin 2t \end{pmatrix} + k_2 e^{4t} \begin{pmatrix} \sin 2t \\ \cos 2t + \sin 2t \end{pmatrix}.$$ 

(b) Using the initial condition, we have

$$k_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

and thus $k_1 = 1$ and $k_2 = 0$. The desired solution is

$$Y(t) = e^{4t} \begin{pmatrix} \cos 2t \\ \cos 2t - \sin 2t \end{pmatrix}.$$ 

(c)

15. (a) In the case of complex eigenvalues, the function $x(t)$ oscillates about $x = 0$ with constant period, and the amplitude of successive oscillations is either increasing, decreasing, or constant depending on the sign of the real part of the eigenvalue. The graphs that satisfy these properties are (iv) and (v).

(b) For (iv), the natural period is approximately 1.5, and since the amplitude tends toward zero as $t$ increases, the origin is a sink. For (v), the natural period is approximately 1.25, and since the amplitude increases as $t$ increases, the origin is a source.

(c) (i) The time between successive zeros is not constant.
   (ii) Oscillation stops at some $t$.
   (iii) The amplitude is not monotonically decreasing or increasing.
   (vi) Oscillation starts at some $t$. There was no prior oscillation.